1. Introduction

It can be safely said that optical soliton pulses are one of the most thoroughly investigated and well-understood phenomena in nonlinear photonics. Since the seminal works of Hasegawa and Tajkhorshid [1] and later the experimental observations of F. H. Taylor and collaborators [2], the evolving helmholtz equation (SVEA) and the emerging gallery boost to a local time frame have been the natural frame of reference for the analysis of phenomena described by the SVEA. A review of the salient points is that

\[ \frac{\partial^2 u}{\partial t^2} + \alpha^2 \nabla^2 u + \beta \nabla^4 u - \gamma u = 0 \]

where \( u \) denotes the output power of the SVEA.\footnote{A. Hasegawa and F. Tajkhorshid, Appl. Phys. Lett. 23, 142 (1973).} The above equation describes the propagation of light in a nonlinear medium.\footnote{P. L. McRobbie and J. P. Gordon, Opt. Lett. 8, 119 (1983).} The parameter \( \alpha \) is the refractive index of the core medium at zero frequency, \( \beta \) is the vacuum speed of light, and \( \gamma \) is the nonlinear coefficient of the medium.\footnote{R. W. Boyd, Nonlinear Optics, 2nd Ed., Academic Press, 2003.}

The natural question to ask is, what happens if one keeps the \( n^2 \) constant while implementing the Gaussian boost?\footnote{F. Biancalana and C. C. Crebret, Opt. Exp. 16, 14682 (2008).} In that case, the governing equation on a cross-domain operator which can hinder a straightforward physical interpretation:

\[ \frac{\partial^2 u}{\partial t^2} + \alpha^2 \nabla^2 u + \beta \nabla^4 u - \gamma u = 0 \]

To proceed, one must, in fact, consider only those families of solutions where \( \alpha^2 \) is constant, which is the case, for example, on order of magnitude considerations, for omitting the cross-domain term. In so doing, one ends up with the approximate model of Biancalana and Crebret [4], which is a temporal analogue of the classical helmholtz equation [6].

2. Helmholtz Pulse Model

When considering problems involving pulse propagation, typically a transverse electric field is assumed, and a scalar electric field is obtained.\footnote{J. M. Christian, G. S. McDonald, T. F. Hodgkinson, and P. Chamorro-Posada, J. Mod. Opt. 55, 1216 (2008).} The group velocity is defined as

\[ \vec{V}_g = \frac{1}{n^2} \frac{\partial n}{\partial \omega} \vec{F} \]

where \( n \) is the refractive index of the core medium at zero frequency, \( \omega \) is the frequency, and \( \vec{F} \) is the electric field.\footnote{J. M. Christian, G. S. McDonald, T. F. Hodgkinson, and P. Chamorro-Posada, J. Mod. Opt. 55, 1216 (2008).}

2. Local-Time vs. Laboratory Frames

When investigating the properties of pulse propagation, typically a transverse electric field is assumed, and a scalar electric field is obtained.\footnote{J. M. Christian, G. S. McDonald, T. F. Hodgkinson, and P. Chamorro-Posada, J. Mod. Opt. 55, 1216 (2008).} The group velocity is defined as

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3. Velocity Combination Rule

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4. Soltions & Relativity

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