Modelling warranty extensions: a case study in the automotive industry

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ABSTRACT: Warranty extensions are considered for automotive vehicles. In particular, the expected cost to a manufacturer of an extension to a base warranty is determined, while taking account of the effect of services during the warranty period. To model the effect of services, we use an inspection maintenance model that is based on the delay time concept. Therein, failures are preceded by defects, and the inspection of a vehicle while in the defective state facilitates the correction of such defects. Thus we model the typical circumstances in which a warranty places certain requirements on the customer to carry out a standard level of servicing. We regard a vehicle as a complex system, with defects arising according to a counting process. We also allow defects to be corrected when a vehicle fails. A case study carried out for Malaysian Truck Berhad is used to illustrate these concepts.

KEYWORDS: Warranty; maintenance; delay time modelling.

1. Introduction

This paper is concerned with modelling the cost of warranty extensions for a particular automotive supplier in Malaysia. The supplier is concerned with the cost of extending the base warranty that is offered to vehicle owners on a new purchase from the supplier. Many motor manufacturers are now offering warranties on new vehicles of three and five years in length, and Malaysian Truck Berhab (MTB) are seeking to compete in the automotive sales market using a similar strategy. The HICOM Perkasa MTB160 in particular is widely used in Malaysia, especially in small-medium enterprises. MTB provides a base warranty over 12 months or 20,000 km mileage for this vehicle. Although the commercial set-up is rather more complex than we state here, for the purpose of our study, we shall suppose that MTB wish to consider the likely cost of extending the base warranty period for this vehicle.

Much of our detailed results may be regarded as specific to the client; however, we make a number of general contributions. The first concerns the modelling of warranty claims. Inspections occur in the warranty period; we model the warranty claims and inspections (services) using a delay time modelling approach (Christer, 1999). We consider the system as a complex one and use

a Poisson process to model claims, and allow defects to be repaired at failure events as well as on detection at inspection. The application of this model to automotive warranty claims is novel, as far as we are aware. The second is to the calculation of expected costs based on this model. While costing failures alone may be developed using renewal theory, determination of costs of defects and failures is more difficult. We also make a number of interesting observations. The first is that careful definition is required of what constitutes a failure and what constitutes a defect so that those responsible for data collection across the organization are acting consistently and also so that the costs implications can be appropriately interpreted. Secondly, it appears that the rate of arrival of warranty claims for an individual vehicle is broadly speaking a constant, (after an initial wear-in period) over a time scale (for the individual vehicle) that is not too long. The consequences of this for the cost of a warranty extension are then important; costs will not tend to grow out of control. Also, from a technical point of view, the expected cost will be less difficult to calculate in principle because after the wear-in period the claim rate will be constant. Of course, one can always resort to simulation for the calculation of expected cost when mathematical analysis becomes intractable. Good knowledge of input parameters (defect and failure rates) will be very useful for such simulation work.

Warranty analysis has received significant attention in the recent past (e.g. Blishke and Murthy, 1996; Murthy and Blischke, 2006). Many works are theoretical in nature and discuss numerous types of warranty policy, from the straightforward fixed non-renewing warranties (warranty valid from 1 year from purchase) through to complex agreements between warranty parties. Studies based on real data have also appeared (e.g. Attardi et al, 2005). Studies relating to the automotive industry have been published (e.g. Summit and Cerone, 2003; Iskandar and Blischke, 2003; Rai and Singh, 2004). Our approach is different in that we consider inspection policy during the warranty period and we model inspections through the concept of the delay time (Christer, 1999). The cost analysis of a warranty can be viewed from different perspectives: that of the manufacturer, that of the buyer or consumer; and that of a third party (Chattopadhyay and Murthy, 2000). The last of these relates mainly to extended warranties which we do not consider. From the point of the view of a buyer, a base warranty extension may not be all that in seems. Typically, a manufacturer will require servicing to be carried out by an approved facility in order for the warranty to remain valid. Servicing and planned maintenance at approved facilities can cost a premium over non-registered facilities. This can make base warranty extensions more affordable to manufacturers provided contractual agreements with service providers are in place (see Murthy and Blishke, 2006, p.226). We are specifically interested here in the cost to the manufacturer of a base warranty extension. While such warranty extensions may increase sales this aspect of cost analysis is beyond our scope, although one might, in a grand decision model, take account of such a benefit.

Thus, in summary, we model warranty claims and the inspection or servicing process. We do this in section 3 of the paper. This is preceded by an exploratory analysis of the warranty claims data for the particular vehicle of interest. The model developed in section 3 is fitted to data in section 4. In section 5, cost estimation for the base warranty extension is developed, and cost

calculations are illustrated based on the model we fit to the claim and inspection data. Finally, we then make some recommendations both in particular and in general.

2. Exploratory analysis of the warranty claims data

The data we describe relate to the HICOM Perkasa MTB160. In total 8134 vehicles were sold in Malaysia over the period 2000 to 2004, and we have information about warranty claims for these vehicles. Furthermore, we have more detailed data relating to service history during the warranty period for a sample of 101 of these vehicles sold in 2003 and 2004. The vehicles were serviced by ACM, a servicing agent for MTB in Malaysia. The vehicle histories had been the subject of earlier analysis by ACM and were chosen for their richness. This, unfortunately, implies that this sample cannot be considered as a random sample, and in fact, the average claim rate is remarkably high for this sample.

Claims data are collected for all HICOM Perkasa products by MTB from distributors, and among the information provided are: date claim received; date claim made; failure date; sale date; production date; mileage; model; part (component) type; engine number; and registration number. This allows us to construct a chart that classifies claims, figure 1. Not all claims are accepted under the warranty, figure 2. The MTB160 has the largest number of claims; we concentrate on this model for this reason. Figure 3 shows that electrical faults account for approximately 40% of warranty claims of the MTB160. The age of the product at the time of claim was calculated for each claim; a histogram of age at claim is shown in figure 4, along with a fitted Weibull distribution. This appears to be a good fit. The average age at warranty claim is 130 days. The histogram of usage (kilometers traveled since new) with fitted Weibull (9643,0.747) distribution is shown in Figure 5. The average usage at warranty claim is 12058 kilometres.

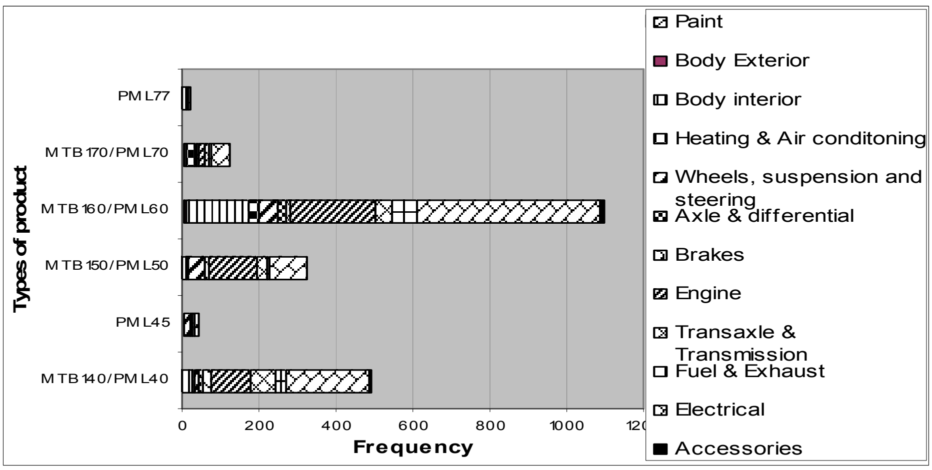

Figure 1. Number of warranty claims for each type of HICOM Perkasa product by component (2000 to 2004)

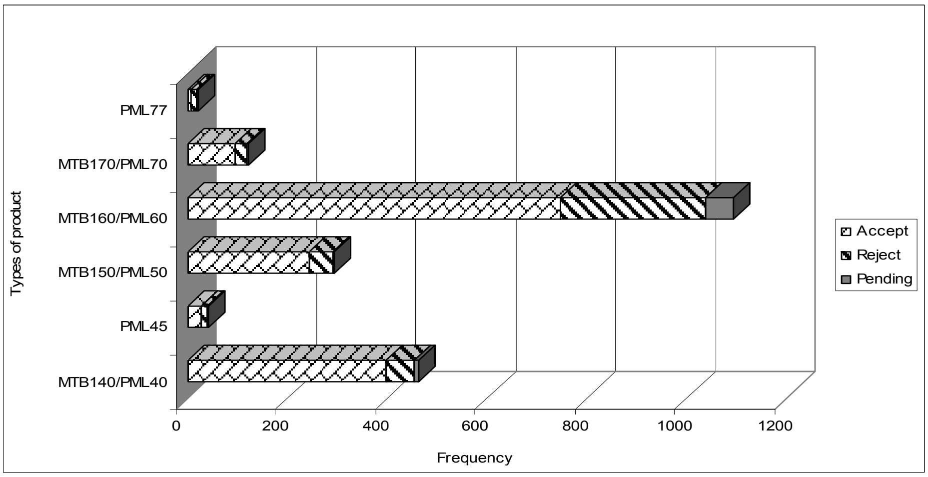

Figure 2. Claim status for HICOM Perkasa products (2000 to 2004).

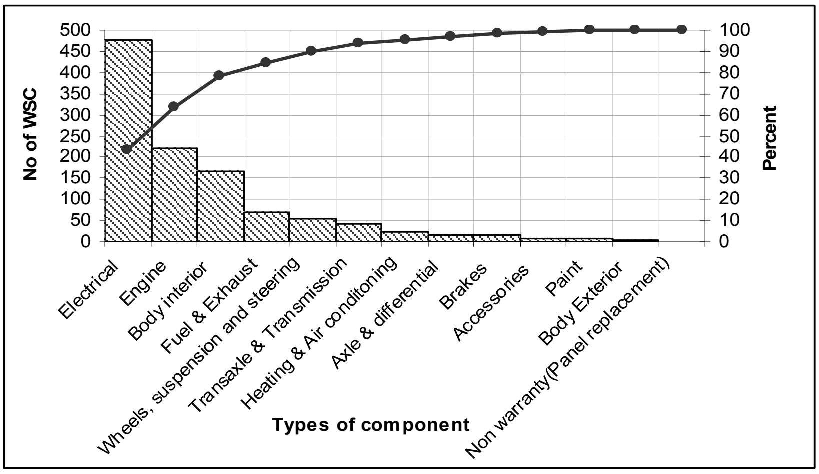

Figure 3. Pareto chart of warranty claims for the 8134 HICOM Perkasa MTB160 vehicles sold over 2000 to 2004.

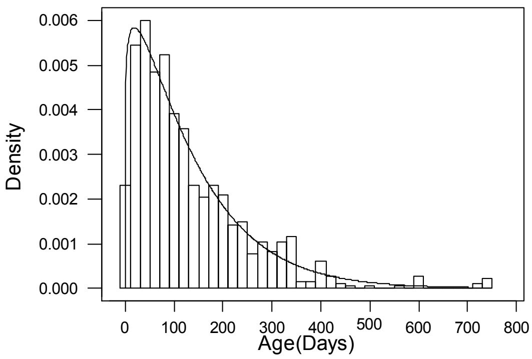

Figure 4. Histogram of age at warranty claim for the 8134 HICOM Perkasa MTB160 vehicles sold over 2000 to 2004, with fitted Weibull distribution, $F(y) = 1 - \exp\{-(y/\alpha)^{\beta}\}$; ($\alpha = 135.8$ and $\beta = 1.117$).

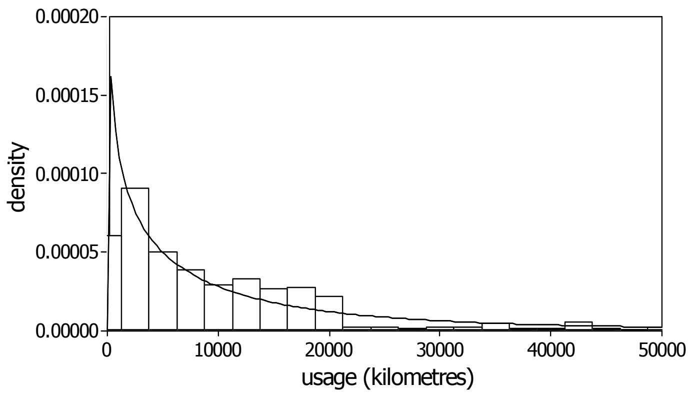

Figure 5: Histogram of usage at warranty claim for the 8134 HICOM Perkasa MTB160 vehicles sold over 2000 to 2004, with fitted Weibull (9640, 0.747) distribution.

The expected number of claims per unit at age *a*, denoted by $\hat{\lambda}(a)$, can be calculated using the method of Kalbleisch and Lawless(1996):

$$\hat{\lambda}(a) = \frac{n_T(a)}{R_T(a)}, \quad a = 0,1,2.........,T,$$

where $n_T(a) = \sum_{d=0}^{T-a} n(d,a)$ is the total number of age a claims occurring up to month T, $n(d,a)$ is the number of claims at age a for the units sold in month d, and $R_T(a) = \sum_{d=0}^{T-a} N(d)$ is the total number of units that have reached age a on or before month T. The standard error of the estimated claim rate can also be obtained.

The expected number of claims $\hat{\lambda}(a)$ is high during the initial stage of product life (up to age 12 months), see figure 6. The maximum expected number of claims is at 2 months old with 0.0192 claims per vehicle. $\hat{\lambda}(a)$ is decreasing monotonically from age 24 months onward. The final value for the cumulative rate per vehicle is $\hat{\Lambda}(60) = 0.121$ where $\Lambda(a) = \sum_{u=0}^{a} \lambda(u)$. This implies there are on average per vehicle 0.121 claims over the first 5 years of the life of a vehicle. This is considered a low rate for an automotive product. One can compare this figure with the crude measure of claim rate given by the ratio of number of claims to number sold (=1097/8134=0.135).

For the 101 vehicles in the detailed sample with service histories, there were 130 warranty claims over 2003 and 2004. These vehicles were the subject to high usage: 40,000 km per year on average with some subject to more than 100,000 km in their first year. The service histories consisted of records relating to each of the three free services (provided by ACM under the warranty contract) and subsequent services. There was considerable variation in the regularity of the services and in the information recorded; broadly speaking, records related to faults found and repaired both at services and at unplanned maintenance following breakdowns.

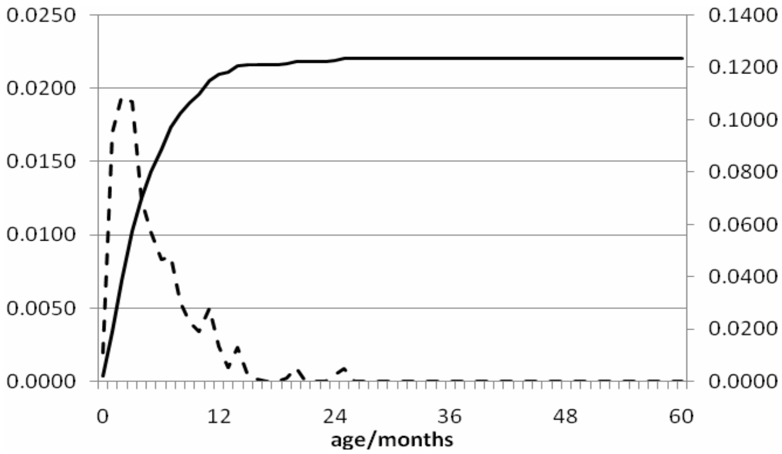

Figure 6. Claim rate per vehicle $\hat{\lambda}(a)$ (----- left hand scale) and cumulative claim rate per vehicle $\hat{\Lambda}(a)$ (—— right hand scale) for MTB160 (2000 to 2004).

3. A delay time model of servicing and warranty claims

Exploratory analysis of the warranty claims data indicated to us that the system (vehicle) has a number of characteristics. Vehicle inspections are scheduled on a periodic basis, but that actual inspections occur aperiodically. Defects may be found at inspection and there may be more than one defect found at inspection. Defects are generally repaired at inspection, and these defects are the subject of a warranty claim if the repair occurs during the warranty period (and the claim is accepted). Failures of the operational function of the vehicle may occur and such failures are often preventable through inspection while the vehicle is in the defective state but is not failed. The definition of a failure can be ambiguous, and requires careful explanation to the problem owners. By failure we shall mean we shall mean an event that requires the vehicle to undergo immediate corrective maintenance; we might refer to this as a breakdown. A failure would normally incur an additional cost to the manufacturer or to the customer or both; this additional cost might be of the nature of a penalty cost (see Christer and Scarf, 1994). On failure, inspection of the vehicle would normally take place during repair; other defects would then be corrected. Failures give rise to a warranty claim if they occur within the warranty period and they satisfy the requirements of a warranty claim. We do not discuss these requirements in detail but merely note that not all claims are accepted and "paid-out". The relationship between claims and actual payouts is complex (see Ward and Christer, 2005, for example), but need not concern us for the purpose of our modelling here; our models will be concerned with accepted warranty claims. We have information about vehicle histories both during and subsequent to the warranty period; such information may be incomplete e.g. vehicle inspection information may be missing particularly if the vehicle is outside the warranty period and service takes place in an un-registered facility.

These considerations suggest that we use a failure model for which the following assumptions hold:

1. defects arise according to some counting process, and we will use the Poisson process for convenience (Ascher and Feingold, 1984);
2. a defect causes a failure after some delay time that is itself a random variable;
3. on inspection, defects are corrected and corrected defects are subject to a warranty claim if the defect is corrected within the warranty period;
4. at a failure repair, the vehicle is inspected.

Thus we consider the system as a multi-component one, so that multiple concurrent defects are possible. Then, failure of one component leads to repair, but other defects present at the repair point may also be repaired. A model for such a system was developed by Christer and Wang (1995) for a manufacturing plant subject to inspection maintenance; we use the model in a warranty context. Early simple delay time models of a multi-component system are different in that they only allowed for defect repair at inspection (Baker and Wang, 1991).

We consider three forms for the defect arrival rate which we denote by $\lambda(u)$: constant arrival rate; asymptotically constant; and power law defect arrival rate. Further, we consider two forms for the delay time distribution with probability density function $f(h)$ and distribution function $F(h)$: exponential and mixed exponential. The mixed exponential distribution has probability mass p on the outcome “zero delay time”. The remaining probability 1-p is spread over the interval $(0,\infty)$. In this way, we model defects that cause immediate failure. Thus a proportion p of defects cause immediate failure; for such defects inspection can have no benefit. In a sense, this idea models imperfect inspection while having the distinct advantage that inspection points are system renewals. For a recent discussion of modelling developments in inspection maintenance, see Wang et al. (2010). Assumptions relating to cost considerations are discussed later in the paper.

Conceptually our failure and inspection model looks like figure 7.

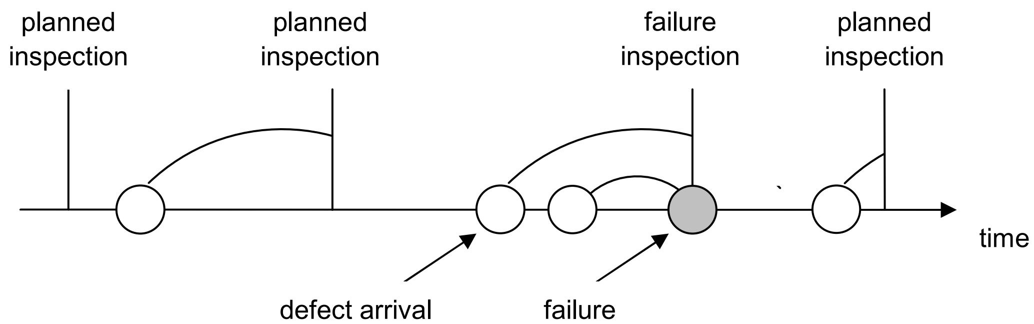

Figure 7. Conceptual delay time model with aperiodic planned inspections and inspection on failure.

We have to also make a number of technical assumptions to allow the model to be tractable. These are as follows.

5. The delay time is independent of the defect arrival process.

6. Inspections are perfect; all defects present at inspection are repaired; the process of inspection does not have any effect on the defect arrival process.
7. Inspections and repairs are instantaneous.

Assumption 6 simplifies the model fitting considerably. Baker et al (1997) describe a more complex model in which inspections may be imperfect. This model has two major drawbacks. Firstly, the likelihood function is rather difficult to determine; secondly, long delay-times and perfect inspection are difficult to disaggregate because both can lead to circumstances in which few failures occur and many defects are found at inspection. Thus the model is rather demanding of data.

With a fully specified model, we proceed in two steps. First we fit this model to the vehicle maintenance history data to obtain the values of the parameters of the model. Then, having determined the values of the cost parameters, we use the failure and inspection model to determine the cost of warranty claims over a warranty extension period. Model fitting is considered in the following section. In our analysis we consider the vehicle model HICOM Perkasa MTB160 only so that actual results are applicable to this vehicle. The methodology itself has wider applicability.

4. Model fitting

Let t and s be two consecutive inspection times ($t > s$). We use $(s_{\mathrm{F}}, t_{\mathrm{F}}]$ to denote a failure interval; that is an interval in which the final event is a failure at time t_{F}. Let m_{F} be the additional number of defects that are identified at the inspection that takes place at t_{F}. We use $(s_{\mathrm{I}}, t_{\mathrm{I}}]$ to denote an inspection interval where a planned, service inspection takes place on an otherwise working system at time t_{I}. Let m_{I} be the number of defects identified at the inspection at time t_{I}. A vehicle history is then a sequence of intervals which themselves may be failure or inspection intervals. Suppose that there are n_{F} failure intervals and n_{I} planned inspection intervals in a vehicle history (of an individual vehicle). Following our modelling assumptions above (in particular perfect inspection and that defect arrivals occur according to a Poisson process) it follows that all intervals are (statistically) independent—or more specifically events occurring in non-overlapping intervals are independent.

With this notation in mind, it follows that a vehicle history consists of the following data: failure times t_{F_i}, the times of the corresponding previous inspections s_{F_i}, and the number of defects found at the failure inspections m_{F_i} for $i = 1,\ldots,n_{\mathrm{F}}$; planned inspection times t_{I_i}, the times of the corresponding previous inspections s_{I_i}, and the number of defects found at the planned inspections m_{I_i} for $i = 1,\ldots,n_{\mathrm{I}}$.

From Christer and Wang (1995), we have that the probability of m_{F} defects found at a failure inspection is

$$\frac{1}{(m_{\mathrm{F}}+1)!}\exp\{-\int_{s_{\mathrm{F}}}^{t_{\mathrm{F}}}\lambda(u)du\}.\int_{s_{\mathrm{F}}}^{t_{\mathrm{F}}}\lambda(u)f(t_{\mathrm{F}}-u)du.\left[\int_{s_{\mathrm{F}}}^{t_{\mathrm{F}}}\lambda(u)\overline{F}(t_{\mathrm{F}}-u)du\right]^{m_{\mathrm{F}}}.$$

This follows because defects arrive according to an Poisson process with rate $\lambda(u)$, and as this considers a failure interval, there was one defect that caused the failure giving the contribution

$$\int_{s_{\mathrm{F}}}^{t_{\mathrm{F}}} \lambda(u) f(t_{\mathrm{F}} - u) du ,$$

noting that the failure itself occurs at time t_{F}. A further m_{F} defects arise; these latter defects each do not cause a failure but survive to time t_{F}. This then gives the contribution

$$\left[\int_{s_{\mathrm{F}}}^{t_{\mathrm{F}}} \lambda(u) \overline{F}(t_{\mathrm{F}} - u) du\right]^{m_{\mathrm{F}}} .$$

Here $\overline{F}(x) = 1 - F(x)$ is the probability that a defect survives to time *x*.

For a planned inspection interval the probability of m_{I} defects found at inspection is

$$\frac{1}{m_{\mathrm{I}}!} \exp\{-\int_{s_{\mathrm{I}}}^{t_{\mathrm{I}}} \lambda(u) du\}.\left[\int_{s_{\mathrm{I}}}^{t_{\mathrm{I}}} \lambda(u) \overline{F}(t_{\mathrm{I}} - u) du\right]^{m_{\mathrm{I}}}$$

as all defects arising in the interval survive to inspection.

The log likelihood function for the data under our model, with n_{F} failure intervals and n_{I} planned inspection intervals for a single vehicle (vehicle *k,* say), is then

$$\begin{aligned} L_k = \sum_{j=1}^{n_{\mathrm{F}}} & \left\{ -\log(m_{\mathrm{F}_j} + 1)! - \int_{s_{\mathrm{F}_j}}^{t_{\mathrm{F}_j}} \lambda(u) \mathrm{d}u + \log\left[\int_{s_{\mathrm{F}_j}}^{t_{\mathrm{F}_j}} \lambda(u) f(t_{\mathrm{F}_j} - u) \mathrm{d}u\right] + m_{\mathrm{F}j} \log\left[\int_{s_{\mathrm{F}_j}}^{t_{\mathrm{F}_j}} \lambda(u) \overline{F}(t_{\mathrm{F}_j} - u) \mathrm{d}u\right]\right\} \\ & + \sum_{i=1}^{n_{\mathrm{I}}} \left\{ -\log(m_{\mathrm{I}_i}!) - \int_{s_{\mathrm{I}_i}}^{t_{\mathrm{I}_i}} \lambda(u) \mathrm{d}u + m_{\mathrm{I}i} \log\left[\int_{s_{\mathrm{I}_i}}^{t_{\mathrm{I}_i}} \lambda(u) \overline{F}(t_{\mathrm{I}_i} - u) \mathrm{d}u\right]\right\}. \end{aligned} \quad (1)$$

With multiple vehicles, assuming all vehicles are independent of one another, one can determine the log-likelihood contribution for each vehicle. A number of approaches are now possible. If we further assume that all vehicles are (statistically) identical then the overall likelihood is just $L = \sum_{i=1}^{k} L_k$. One might instead model vehicles so that they are statistically different from each other, through $\lambda(u)$ or *f* or both. How one proceeds will in practice be determined by how much data are available. In our case data are limited so we take the simple approach assuming vehicles are statistically identical. An interesting possibility might be to use a random effects model (e.g. Crowder and Lawless, 2007).

One might further suppose that λ is mileage related. Denoting the mileage of a vehicle by *z* we might model $\lambda = \lambda(u, z)$ in the manner proposed by Moskowitz and Chun (1994). A simpler approach would be to suppose $\lambda(u)$ is related to average annual mileage—vehicles with a high annual mileage will have a large $\lambda(u)$. In fact we might make $\lambda = \lambda(u, \bar{z})$. If $\bar{z}_k$ is the annual average mileage of vehicle *k*, in a model for which λ is constant (with age) but vehicle specific, we might propose $\lambda = \lambda \times (\bar{z}_k / \bar{z}_A)$ where $\bar{z}_A$ is the average annual mileage over all vehicles (or the average annual mileage of the idealized vehicle e.g. 20K km per annum). The cost calculations (considered later) could then be considered "two-dimensionally", based on age and mileage. We might proceed similarly if λ is a baseline parameter in an age-varying defect arrival rate. We consider these points for future development.

In our model, it now remains to specify the defect arrival rate $\lambda(u)$ and the delay time distribution $f(h)$. We use a number of alternatives. For the defect arrival rate, we use

$$\lambda(u) = \lambda \quad \text{(constant)},$$

$$\lambda(u) = \frac{\lambda_0 + \lambda_1 u}{1+u} \quad \text{(asymptotically constant)},$$

$$\lambda(u) = \lambda \delta u^{\delta - 1} \quad \text{(power law)}.$$

The second of these has a rather simple interpretation: the initial defect rate is λ_0, the final defect rate ($u \to \infty$) is λ_1 and the defect rate at 1 time unit old is $\frac{1}{2}(\lambda_0 + \lambda_1)$. A faster rate of decay (or increase) may be obtained by raising u to some power. For the delay time distribution we use the exponential distribution $f(x) = \gamma e^{-\gamma x}$ and the mixed exponential distribution

$$f(x) = \begin{cases} p & x = 0, \\ (1-p)\gamma e^{-\gamma x} & x > 0. \end{cases}$$

With the mixed exponential we have to be little careful in the calculation of f and $\overline{F}$ in the likelihood (equation 1). This becomes

$$\begin{aligned} L_k = \sum_{j=1}^{n_\mathrm{F}} & \left\{ -\log(m_{\mathrm{F}_j} + 1)! - \int_{s_{\mathrm{F}_j}}^{t_{\mathrm{F}_j}} \lambda(u)\mathrm{d}u + \log\left[p\lambda(t_{\mathrm{F}_j}) + (1-p) \int_{s_{\mathrm{F}_j}}^{t_{\mathrm{F}_j}} \lambda(u)\beta e^{-\beta(t_{\mathrm{F}_j} - u)} \mathrm{d}u \right] \right\} \\ & + \sum_{j=1}^{n_\mathrm{F}} \left\{ m_{\mathrm{F}_j} \log\left[(1-p) \int_{s_{\mathrm{F}_j}}^{t_{\mathrm{F}_j}} \lambda(u) e^{-\beta(t_{\mathrm{F}_j} - u)} \mathrm{d}u \right] \right\} \\ & + \sum_{i=1}^{n_\mathrm{I}} \left\{ -\log(m_{\mathrm{I}i}!) - \int_{s_{\mathrm{I}_i}}^{t_{\mathrm{I}_i}} \lambda(u)\mathrm{d}u + m_{\mathrm{I}i} \log\left[(1-p) \int_{s_{\mathrm{I}_i}}^{t_{\mathrm{I}_i}} \lambda(u) e^{-\beta(t_{\mathrm{I}j} - u)} \mathrm{d}u \right] \right\}. \end{aligned} \tag{2}$$

The likelihood in equation (2) above is sufficiently general for our purpose and is maximized using a general function maximizer (using a programme written in Fortran). The Hessian matrix is determined so that estimates of parameter variances (standard errors) can be obtained. The likelihood can be generalized to deal with left censoring (vehicles with unknown history up to the point they enter the study) and right censoring (vehicles with unknown history beyond the end of the study). The vehicle histories we consider in detail finish with an inspection and are therefore uncensored.

4.1. *Model fitting results*

The delay time model of inspection and failure was fitted to the 101 vehicles with full vehicle history. Six forms of the model were considered. The AIC (Sakamoto et al. 1986) for each of the six models is shown in Table 1 and parameters estimates with standard errors for each model are shown in Table 2. The unit of time was one day, so that in the highlighted model, the initial defect arrival rate is 5.99 per year or 1 defect every 2 months.

Table 1. Akaike Information Criterion (AIC) statistic (Sakamoto et al., 1996) for each of the six fitted models. Minimum AIC model highlighted.

Model	AIC
Constant λ, exponential delay time	4394.70
Constant λ, mixed exponential delay time	4290.30
Asymptotic λ, exponential delay time	4275.92
Asymptotic λ, mixed exponential delay time	4173.40
Power law λ, exponential delay time	4362.24
Power law λ, mixed exponential delay time	4343.56

Table 2. Maximum likelihood estimates of parameter (standard errors) for each of the six fitted models. Minimum AIC model highlighted. The time unit here is 1 day.

Model	Parameters				
	P	β	λ_0	λ_1	δ
Constant λ, exponential delay time	N/A	0.00721 (0.00518)	0.0244 (0.0172)	N/A	
Constant λ, mixed exponential delay time	0.144 (0.132)	0.00714 (0.00357)	0.0317 (0.0314)	N/A	
Asymptotic λ, exponential delay time	N/A	0.00911 (0.00517)	0.0201 (0.0162)	0.00781 (0.00322)	N/A
Asymptotic λ, mixed exponential delay time	0.0291 ($2.51*10^{-4}$)	0.00892 ($5.88*10^{-4}$)	0.0164 ($6.41*10^{-4}$)	0.00712 ($3.30*10^{-4}$)	N/A
Power law λ, exponential delay time	N/A	0.0139 (0.0118)	0.00245 (0.00201)	N/A	0.9998 ($5.18*10^{-4}$)
Power law λ, mixed exponential delay time	0.426 (0.241)	0.107 (0.101)	0.0261 (0.0172)	N/A	0.9998 ($5.18*10^{-4}$)

5. Cost analysis of a warranty extension

We are concerned here with the expected cost to the manufacturer of a base warranty extension. At MTB vehicle owners do not pay for the first three services during the warranty period, and it is compulsory for all vehicle owners to have the first three services performed in the manner described in the vehicle manual. Subsequent services require payment. We will formulate the problem more generally, allowing the decision maker control over both the service frequency in the warranty extension and the number of free services. Thus for a base warranty extension, the manufacturer has three decisions to consider. How long should the base warranty extension period be? How many services should be carried out in the base warranty extension period? How many services should be paid for by the manufacturer?

We set the problem up in the following way. First assume that the base warranty extension period is an integer multiple of the base warranty period, and let the base warranty period be the unit of length. Let the base warranty extension period be of length W, and denote the base warranty

extension interval by (1,1+W]. Thus the existing base warranty ends at time t=1. Let K be the number of services (planned inspections) in the base warranty extension, and further assume that the final inspection occurs at time t=1+W. Let K_0 services be paid-for by the customer, and K_1 services be free to the customer ($K = K_0 + K_1$) during the base warranty extension. It follows that the inspection interval during the base warranty extension is $T = W / K$ (K>0). We assume further that an inspection takes place at t=1 (at the end of the base warranty period).

Define the basic costs as follows. Let C_{F} be the cost of a failure (including the failure repair), let C_{D} be the cost of a defect repaired at inspection (failure or planned inspection), and C_{S} the cost of a service. These we take to be constant.

Now if λ is constant (which is reasonable for the asymptotically constant model) we need only calculate the expected total cost over the first planned inspection interval, as subsequent intervals are identical. Let the expected total cost over the first planned inspection interval $(1,1+T]$, excluding the cost of inspections, be $C(T)$. The total expected cost over the base warranty extension interval is then just $K \times C(T) + K_1 C_{\mathrm{S}}$. In terms of the three quantities of interest, W, K and K_0 the expected total warranty cost is

$$C_W = K \times C(W / K) + K_1 C_{\mathrm{S}}, \quad K = K_0 + K_1$$

Here K can be viewed as a decision variable, and for a given (W, K_0) C_{W,K_0} can be minimized with respect to K to give optimum value $\{C^*_{\mathrm{W},K_0}(W, K_0), K^*(W, K_0)\}$. Then the manufacturer can explore values of $\{C^*_{\mathrm{W},K_0}(W, K_0), K^*(W, K_0)\}$ for various values of W and K_0. Note, the service costs might be set up in the converse way, so that a variable number of services are paid-for by the customer. This would imply that the expected total cost is decreasing in K_0. Thus, from the point of view of the manufacturer, it is cost-optimal to insist on servicing "infinitely often" (provided $C_{\mathrm{F}} > C_{\mathrm{D}}$). Of course, this is not a practical course of action. Manufacturers can only insist on the customer paying for a reasonable number of services during the warranty period.

Now $C(T)$ is rather difficult to calculate. This is because, under the policy that we model, inspections are carried when a failure occurs. Thus failures are "renewal points" in addition to planned inspections. When defects are repaired only at planned inspections, the expected total cost over (1,1+T] (excluding the cost of services) is

$$C_{\mathrm{F}} \int_0^T \lambda F(T-u)\mathrm{d}u + C_{\mathrm{D}} \int_0^T \lambda \overline{F}(T-u)\mathrm{d}u \,. \tag{3}$$

The first term is the product of the cost per failure and the expected number of failures in (1,1+T], the second term is likewise for defects. For the situation we are concerned with, we can attempt to find an approximation to $C(T)$ in the following way. Let p_{ij} be the probability of i defects and j failures in (1,1+T]. Note $i \geq j$ since a failure must be preceded by a defect. Now

$$p_{00} = \exp(-\lambda T) \,.$$

This is just the probability of zero defects in (1,1+T], and no defects necessarily implies no failures. Now, conditional on a defect arising at time $X_1 = x_1$, it survives to time 1+T with probability $\overline{F}(T - x_1)$, where F is the delay time distribution (cdf). Further, as $X_1 \sim U(0,T)$ (a

property of the Poisson process), the probability density function g_X of X_1 is just $1/T$. Therefore, if a defect arises the probability that it survives to $1+T$ is

$$\int_0^T \overline{F}(T-x_1)g_X(x_1)\mathrm{d}x_1 = \frac{1}{T}\int_0^T \overline{F}(T-x_1)\mathrm{d}x_1 .$$

Therefore,

$$p_{10} = \exp(-\lambda T).\lambda T.\{\frac{1}{T}\int_0^T \overline{F}(T-x_1)\mathrm{d}x_1\} = \lambda\exp(-\lambda T)\int_0^T \overline{F}(T-x_1)\mathrm{d}x_1$$

Similarly

$$p_{11} = \lambda\exp(-\lambda T)\int_0^T F(T-x_1)\mathrm{d}x_1$$

Thus as a first approximation we have

$$C(T) \approx 0\times p_{00} + C_{\mathrm{D}}\times p_{10} + C_{\mathrm{F}}\times p_{11} = C_{\mathrm{D}}p_{10} + C_{\mathrm{F}}p_{11}.$$

A better approximation would be

$$C(T) \approx C_{\mathrm{D}}p_{10} + C_{\mathrm{F}}p_{11} + 2C_{\mathrm{D}}(1-p_{00}-p_{10}-p_{11}),$$

putting the remaining probability $(1-p_{00}-p_{10}-p_{11})$ on the event "two defects arising in $(1,1+T]$ each surviving to $1+T$".

Now if we allow two defects to arise in $(1,1+T]$, then by developing the argument above further we can improve the approximation. Now conditional on two defects in $(1,1+T]$, there four possible cases illustrated in figure 8. The probability of the first of these (two defects both surviving to $1+T$) is

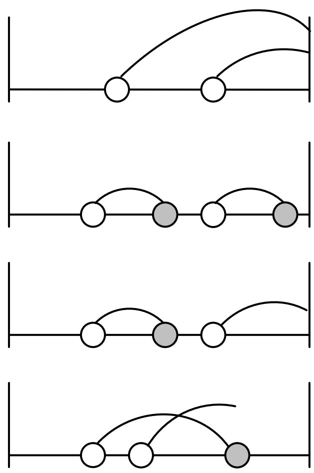

Figure 8: Possible outcomes if two defects arise in a renewal interval.

$$p_{20} = \tfrac{1}{2}(\lambda T)^2\exp(-\lambda T)\left(\frac{1}{T}\int_0^T \overline{F}(T-x_1)\mathrm{d}x_1\right)^2 = \tfrac{1}{2}\lambda^2\exp(-\lambda T)\left(\int_0^T \overline{F}(T-x_1)\mathrm{d}x_1\right)^2.$$

The probability of the second case (two defects both leading to failure) is

$$p_{22} = \tfrac{1}{2}(\lambda T)^2 \exp(-\lambda T) \int_0^T \int_{x_1}^T F(x_2 - x_1) F(T - x_1) g_{X_2}(x_2) g_{X_1}(x_1) \mathrm{d}x_2 \mathrm{d}x_1 ,$$

where X_1 and X_2 are the time of defect arrival for defects 1 and 2 respectively. Note, defect 2 must not arise before defect 2 fails otherwise it will be repaired at inspection on failure of defect 1. Further, g_{X_1} and g_{X_2} are the density functions of the time to first defect arrival and time to second defect arrival respectively. As defects arise according to a Poisson process, X_1 is the first order statistic from a random sample of size 2 from a uniform distribution and X_2 is the second order statistic from a random sample of size 2 from a uniform distribution. Thus $X_1 \sim \mathrm{Be}(1,2,T)$ and $X_1 \sim \mathrm{Be}(2,1,T)$. Here $\mathrm{Be}(\alpha,\beta,T)$ denotes the beta distribution on $[0,T]$ with density function:

$$g(x) = x^{\alpha-1}(T-x)^{\beta-1} / \int_0^T x^{\alpha-1}(T-x)^{\beta-1} \mathrm{d}x ,$$

so that

$$g_{X_1}(x) = 2(T-x)/T^2 ,$$

and

$$g_{X_2}(x) = 2x/T^2 .$$

Thus

$$p_{22} = 2\left(\frac{\lambda}{T}\right)^2 \exp(-\lambda T) \int_0^T \int_{x_1}^T x_2 (T - x_1) F(x_2 - x_1) F(T - x_1) \mathrm{d}x_2 \mathrm{d}x_1 .$$

For the third and fourth cases, the probabilities are

$$\int_0^T \int_{x_1}^T F(x_2 - x_1) \overline{F}(T - x_2) g_{X_2}(x_2) g_{X_1}(x_1) \mathrm{d}x_2 \mathrm{d}x_1 ,$$

and

$$\int_0^T \int_{x_1}^T \int_{x_2 - x_1}^{T - x_1} \overline{F}(h_1 + x_1 - x_2) f(h_1) g_{X_2}(x_2) g_{X_1}(x_1) \mathrm{d}h_1 \mathrm{d}x_2 \mathrm{d}x_1 .$$

In the second of these, h_1 denotes the delay time for the first defect. It follows that

$$p_{21} = 2\left(\frac{\lambda}{T}\right)^2 \exp(-\lambda T) \int_0^T \int_{x_1}^T x_2 (T - x_1) \left\{ F(x_2 - x_1) \overline{F}(T - x_2) + \int_{x_2 - x_1}^{T - x_1} \overline{F}(h_1 + x_1 - x_2) f(h_1) \mathrm{d}h_1 \right\} \mathrm{d}x_2 \mathrm{d}x_1 .$$

Noting that, by a similar argument,

$$p_{30} = \tfrac{1}{6} \lambda^3 \exp(-\lambda T) \left(\int_0^T \overline{F}(T - x_1) \mathrm{d}x_1 \right)^3$$

our improved approximation is

$$C(T) \approx C_{\mathrm{D}} p_{10} + C_{\mathrm{F}} p_{11} + 2C_{\mathrm{D}} p_{20} + (C_{\mathrm{F}} + C_{\mathrm{D}}) p_{21} + 2C_{\mathrm{F}} p_{22} + 3C_{\mathrm{D}} p_{30} .$$

The next term in the approximation $(C_{\mathrm{F}} + 2C_{\mathrm{D}}) p_{31}$ is calculated in the appendix. We might also suppose that the probability not yet accounted for be assigned to p_{32} and the corresponding term $(2C_{\mathrm{F}} + C_{\mathrm{D}}) p_{32}$ added to the cost. In practice, one would consider each of the approximations in turn, and check they converge before the calculations become impractical. Care must be taken with

the evaluation of p_{ij} when using the mixed exponential distribution for the delay times, particularly the second term in p_{21}.

Our experience is that this approximation will be reasonable provided λ is not too large (λ<0.2). Of course, if λ is very large then warranty extension is likely bankrupt the manufacturer. Another approximation might calculate the average number of failures ignoring inspection at failure, r say. Then suppose this number of inspections are carried out in (1,1+T], equally spaced; the expected number of failures and defects arising in (1,1+T/r] and the expected cost are then as in expression (3). Finally, one would need to scale up the cost by the factor r. One might also use simulation.

5.1. *Cost calculation results*

For simplicity then we assume λ constant. Futhermore we use the value from section 2 calculated using the Kalbfleisch-Lawless method. Arguably, the defect arrival rate may be somewhat higher than this value; but we feel that this value is certainly more representative of the vehicle than the value obtained from the 101 vehicle sample. We further assume than the delay time follows a mixed exponential distribution with p and δ obtained from the best fitting delay time model fitted to the 101 vehicle sample (highlighted in table 4). Given the uncertainty regarding parameter estimates then, we must content ourselves that the cost calculations below represent an illustration of the methodology. The methodology in our view is sound; one would need have complete claim and service history for a large, random sample of vehicles in order to obtain useful warranty extension cost estimates.

Results are obtained for warranty extensions of 1, 2 and 3 years (W=1,2, or 3), and 1 and 2 services paid for by the customer each year (if W=1, K_0=1 or 2; if W=2, K_0=2 or 4; etc.). In table 3, we present the expected cost per vehicle per year over the warranty extension period. Figure 9 shows the expected cost per year over the warranty extension, $C_{W,K_0}/W$, as a function of K. We can see that expected cost per unit time does not depend on the length of the warranty extension, when the inspection frequency is constant. This is because the defect arrival rate is constant. Also, as the defect arrival rate is low, inspections are not cost-effective from a failure prevention point of view. Thus, if one were to strictly search for an optimum inspection policy, then K*=0 would be the outcome. Of course, from a vehicle maintenance point of view, this would be unreasonable as routine services (oil changes, fluid level checks) need to be carried out. However, inspecting for the presence of defects is uneconomic.

The analysis here could be extended to the case in which the defect arrival rate is piecewise constant. This would allow for a decreasing defect arrival rate of the sort seen in section 2. The change-points would need to coincide with inspections in order for the formulae relating to the cost calculations above to apply. The formulae above also imply that an inspection is carried out at the end of year 1, regardless of the subsequent policy in the warranty extension period. Relaxing this assumption would be straightforward with regards to the calculation, although one would need then to calculate the expected costs from new, and obtain the costs over warranty extension periods by subtraction.

Table 3. Minimum expected cost per vehicle per year $C^*_{\mathrm{W},K_0}/W$ over the warranty extension, as a function of warranty extension period W, number of inspections K, and number of inspections (services) paid-for by the customer K_0. C_F=570.5, C_D=145.9 and C_S =120.0. Costs in Ringgit Malaysia (1 Ringgit Malaysia = 3.8 US$ approximately).

	W=1			W=2			W=3		
K	K_0=0	K_0=1	K_0=2	K_0=0	K_0=2	K_0=4	K_0=0	K_0=3	K_0=6
1	172.6	52.6		118.8			99.1		
2	286.2	166.2	46.2	172.6	52.6		137.0		
3	421.7	301.7	181.7	227.4	107.4		172.6	52.6	
4				286.2	166.2	46.2	208.8	88.8	
5				350.6	230.6	110.6	246.5	126.5	
6				421.7	301.7	181.7	286.2	166.2	46.2
7							328.4	208.4	88.4
8							373.5	253.5	133.5
9							421.7	301.7	181.7

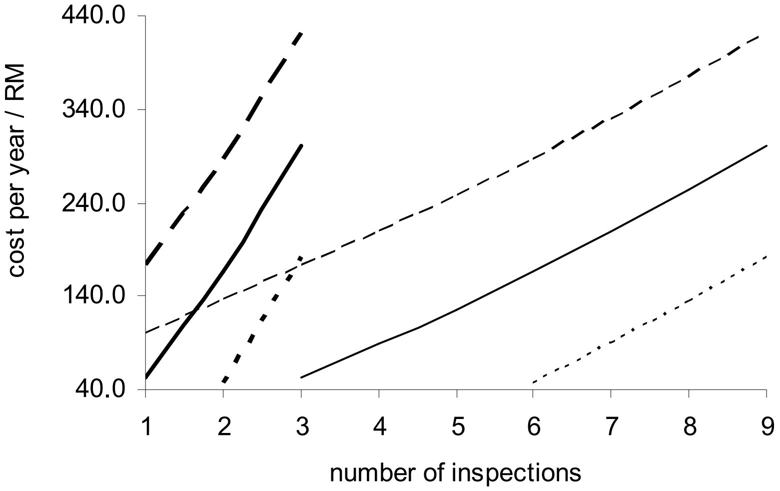

Figure 9. Expected cost per vehicle per year $C^*_{\mathrm{W},K_0}/W$ over the warranty extension as a function of the number of inspections (services) K, for: $W=1$, $K_0=0$ (▬ ▬ ▬); $W=1$, $K_0=1$ (▬▬▬); $W=1$, $K_0=2$ (▪▪▪▪▪); $W=3$, $K_0=0$ (— — —); $W=3$, $K_0=3$ (——); $W=3$, $K_0=6$ (-----). C_F=570.5, C_D=145.9 and C_S =120.0. Costs in Ringgit Malaysia (1 Ringgit Malaysia = 3.8 US$ approximately).

6. Discussion

This paper considers warranty extensions for automotive vehicles. We determine the expected cost to a manufacturer of an extension to a base warranty while taking account of the effect of services during the warranty period. To model the effect of services, an inspection maintenance model based on the delay time concept is used. This model assumes that failures are preceded by defects, and that defects present in the vehicle at inspection are corrected. In this way, we attempt to model the typical circumstances in which a warranty places certain requirements on the customer to carry

out a standard level of servicing, and servicing not carried out to the required standard invalidates the warranty. A vehicle is regarded as a complex system, with defects arising according to a Poisson process. We also allow defects to be corrected when a failure repair is carried out.

We consider in detail a vehicle manufactured in Malaysia by Malaysian Truck Berhad. We explore the effect of the warranty extension period and the number of services paid-for by the customer (warranty contract) on the expected total cost per unit time. Broadly we find that the expected cost per unit time for the warranty extension is not growing out of control. One might also set the service cost premium at a level sufficient to meet the additional cost of the warranty extension. For the warranty extension periods considered, we find that if the manufacturer were to offer services free of charge then it would appear to be uneconomic to do anything other than routine service maintenance. That is, the optimum number of inspections is zero. The results from various models established in this research may be specific to MTB but the models and techniques that have been developed are useful to other automobile warranty providers.

A great deal of effort was expended in collecting the data for this research. However, in fact, the size of the dataset, and the information it contained, were the main limitations of our study. The cost analysis we present here is therefore intended to be illustrative of the methodology. In practice, better estimates of the defect arrival rate, its development with age, and the delay time distribution would be required. Such estimates would need to be based on a much larger and more representative sample of vehicle data. Furthermore, in the data records available to us, failure and defect data were often confused and misclassified. If delay time modelling is to be effective, care must be taken with definition of defects and failures at the time of data collection. In the cost calculation we assumed a constant defect arrival rate, whereas our data analysis suggested a decreasing failure rate from an initially high level. On this basis, our cost calculations are only approximate, although they may be reasonable for large warranty extensions and provided the defect arrival rate is not too large. More precise cost estimates might be obtained through simulation. We also assumed vehicles are statistically identical; one could consider relaxing this assumption using a random effects model, or by accounting for vehicle heterogeneity through usage data. Some ideas in relation to these are discussed in the paper, and would, given sufficient data, make interesting studies. A further limitation is the consideration of the failure process for the system as a whole. In a more refined approach, one might consider components separately. However, when a system is resolved into its component parts, model complexity must be balanced against estimability.

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Appendix

Denoting the defect arrival times by X_1, X_2, and X_3, it follows that $X_1 \sim \text{Be}(1,3,T)$ $X_2 \sim \text{Be}(2,2,T)$ and $X_3 \sim \text{Be}(3,1,T)$ so that

$$g_{X_1}(x) = 3(T-x)^2 / T^3,$$

and

$$g_{X_2}(x) = 6x(T-x)/T^3,$$

and .

$$g_{X_3}(x) = 3x^2 / T^3.$$

Denoting the associated delay times by h_1, h_2, and h_3 we have

$$
\begin{aligned}
p_{31} = {} & \frac{6}{T^6}\lambda^3 \exp(-\lambda T)\int_0^T \int_{x_1}^T \int_{x_2}^T (T-x_1)^2 x_2 (T-x_2) x_3^2 \Big\{ F(x_2-x_1)\overline{F}(T-x_2)\overline{F}(T-x_3) \\
& + \int_{x_2-x_1}^{x_3-x_1} \overline{F}(h_1+x_1-x_2)\overline{F}(T-x_3) f(h_1)\mathrm{d}h_1 + \int_{x_3-x_1}^{T-x_1} \overline{F}(h_1+x_1-x_2) F(h_1+x_1-x_3) f(h_1)\mathrm{d}h_1 \\
& + \int_0^{x_3-x_2} \overline{F}(h_2+x_2-x_1)\overline{F}(T-x_3) f(h_2)\mathrm{d}h_2 + \int_{x_3-x_2}^{T-x_2} \overline{F}(h_2+x_2-x_1)\overline{F}(h_2+x_2-x_3) f(h_2)\mathrm{d}h_2 \\
& + \int_0^{T-x_3} \overline{F}(h_3+x_3-x_1)\overline{F}(h_3+x_3-x_2) f(h_3)\mathrm{d}h_3 \Big\} \mathrm{d}x_3 \mathrm{d}x_2 \mathrm{d}x_1 .
\end{aligned}
$$