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# A study of a two-phase inspection policy for a preparedness system with a defective state and heterogeneous lifetime

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**Abstract:** This paper considers an inspection policy for a single component protection or preparedness system, in which the component arises from a heterogeneous population. At any point in time, the system may be in one of three states, good, defective or failed. The system is only required in an emergency, and in order to ensure high availability of the system on-demand, the system undergoes a sequence of inspections. Inspection determines the system state, so that if a transition from the good state occurs between inspections it is not revealed until subsequent inspection. When a defect or failure is revealed, the component is replaced. At the final inspection the component is replaced. We suppose that a component may be either weak or strong, so that the time in the good state has a distribution that is a mixture. In these circumstances, the efficacy of a two-phase inspection policy, with an anticipated high inspection frequency in early life and low inspection frequency in later life, is considered using availability and cost criteria. The policy is investigated in the context of a valve in a natural gas supply network. If the lifetime distributions in the mixture are quite distinct, then cost savings of the order of 5% can be achieved by using the two-phase policy in place of the simpler single phase policy. Furthermore, only if the mean time in the defective state is small or the required availability is very high does the two-phase policy tend to mimic a burn-in policy.

Keywords: maintenance, replacement, burn-in, mixtures, multi-criteria, stand-by, delay time.

## 1. Introduction

This paper models an inspection policy for a preparedness system with a single component. The system consists of a component in a socket which together provide an operational function. The system is required to function only on demand, for example, in the event of an emergency. The component can be in one of three states: good, defective or failed. The functional status of the system depends on the state of the component and is working if the component is good or

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defective and failed if the component is failed. The state of the component is established only by inspection. The component and hence the system deteriorates over time. We suppose that a component arises from a heterogeneous population; that is, a component may be weak, with a short life, or strong, with long life. In particular, the time in the good state (for the component) we model as a mixture. On inspection of the system, if the component is either defective or failed, then the existing component in the socket is substituted by a new component. In anticipation of age-related wear-out, a planned, preventive replacement of the component takes place, subsequent to the inspection phase. A consequence of our definition of system failure is that the system is unavailable if and only if the component is failed. In these circumstances, we consider the efficacy of an inspection policy that is not periodic. We consider a number of decision criteria including average availability, total cost per unit time including the cost of unavailability, and maintenance cost per unit time in a manner similar to Vaurio [1].

The model we develop extends the work of Cavalcante et al. [2] which considers a two-state failure model in which the component may only be good or failed. It also extends the work of Jia and Christer [3] who consider pure inspection for a three state single component system but for homogeneous components. In the two-state, homogeneous case, the policy is a special case of that introduced by Barlow and Proschan [4]. Some results from the two-state failure case are included for comparison.

Scarf et al. [5] consider preventive replacement for a three-state single component online system when the component arises from a mixed population. The model proposed has two phases—early inspections to mitigate the risk that a component is weak and then longer term replacement during the wear-out phase. Inspection can only be effective if a defective state can exist. The idea is similar to burn-in maintenance [6], but without the requirement for testing prior to entering operation. Our paper here describes an analogous policy, in the sense that there are two phases to the maintenance policy, in the context of a preparedness system.

Typical systems that are used intermittently and have unrevealed failures include military defence systems, medical equipment such as defibrillators, cold stand-by systems, and protection systems such as fire suppressers and alarms. Such systems are often referred to as preparedness systems [7]. In this paper we consider valves in a gas supply network; a pressure safety control valve can be regarded as a preparedness system, for example.

In practice, on inspection a defective state may be observed in which a system is not yet failed but has deteriorated such that a failure may be imminent. On detection of a defect, we suppose that a replacement occurs immediately—such is appropriate for high availability systems. This assumption could be relaxed so that maintenance can be planned, but we do not consider this here. We do not then assume that the failure is detected as soon as it occurs; thus our model is different from the majority of maintenance models that have been studied [8]. Vaurio [1] has argued that not much attention has been paid to the economics of systems in which failures are dormant and detected only by periodic testing or inspection. In relation to Vaurio [1] our approach is broader because we allow in addition the component to be in a dormant, defective state.

The delay time model [9] is used to model the three component states; the delay time model supposes that the times in the good and defective states are independent random variables; the latter is called the delay time. A key aspect of our model and the associated policy is the following. If a component can only be in one of two states, and a component deteriorates over time, then

some unavailability is inevitable unless the component is preventively replaced. However, if the component can be in one of three states, then high availability may be achievable with a pure inspection policy; this is because if inspection takes place while the component is in the defective state then the component is replaced and there is no unplanned unavailability. Therefore, pure inspection may be cost efficient with respect to inspection and replacement. In other words, inspections and replacements have different roles for improving system performance. Preventive replacement aims to anticipate failure without checking functional status; but if a defective state is observable at inspection, then inspections also anticipate failure (as well as checking the functional status), and so preventive replacements may not need to be planned in advance. We have to be careful with our definition of preventive replacement here; by preventive replacement we shall not mean the replacement of a defective component at inspection; by preventive replacement we mean the planned replacement of a component whatever its state at the end of the inspection phase.

Another key facet of our model and associated policy is the impact of the existence of weak and strong components on the efficiency of a maintenance policy. Certainly, age based replacement is not effective when components are heterogeneous; preventive replacement is too late for the weak, too early for the strong. Our purpose is to investigate the impact of component heterogeneity on inspection policies for a preparedness system. We find that some unanticipated effects arise when we allow the existence of a defective state. We return to this point later in the paper. We begin by presenting the policy and our assumptions in detail. A number of decision criteria that might be optimized are determined. We also present the downtime, uptime and cost for the simpler two-state failure model without the defective state. The model and associated policies are illustrated using an example relating to maintenance of pressure regulating valves in a gas distribution network.

## 2. The policy

If one periodically inspects a system whose lifetime arises from a mixed population, at an inspection, one gains information about the system: whether it is good, defective, or failed (this the primary information—essentially this is what the maintainer is most interested in); one also gets partial information about whether the component in the system comes from the sub-population with short lives or from the sub-population with long lives, that is, whether the system is weak or strong. This information is partial because inspection cannot reveal if a system is weak or strong. However, the maintainer would be confident about extending the inspection interval (time between successive inspections) for a system that has passed all inspections to date. This then suggests the following policy: from new, carry out $M_1$ inspections every $T_1$ time units, and then subsequently carry out $M_2$ further inspections every $T_2$ time units; if the component in the socket is either defective or failed on inspection, then the component is replaced; further, preventively replace the component at time $M_1T_1 + M_2T_2$ (since new). $T_1$, $T_2$, $M_1$, and $M_2$, are the decision variables. We might anticipate that ($T_1<T_2$) although this is not a requirement of the policy. A pure inspection policy ( $M_1$ or $M_2 = \infty$ ) is a special case, and may indeed be cost optimal—for reasons discussed above. A single phase inspection and replacement policy is a special case ($T_1=T_2$). Pure replacement (age based replacement) is a further special case ($M_1$=0, $M_2$=1 or $M_1$=1, $M_2$=0).

This policy has the property that it not only determines when to carry out maintenance but it also it helps the decision maker to determine what type of maintenance to carry out. The nature of

the optimum policy, whether inspection or inspection and preventive replacement, or pure preventive replacement, will depend on the particular context and the associated values of model parameters.

We assume the single component system consists of a component and socket which together perform an operational function [10]. By replacement we mean the replacement of the existing component in the socket by a new component. On replacement the system is considered to be new (renewal). The sojourn of a component in the good state, *X,* is a random variable with distribution $F_X(t)$ (and reliability $R_X(t) = (1 - F_X(t))$ and density $f_X(t)$) and the corresponding sojourn in the defective state, the delay time, *H*, is a random variable with distribution $F_H(t)$, and corresponding density $f_H(t)$, independent of *X.* The system is failed if and only if the component failed. We suppose that the distribution of *X* is a mixture: $F_X(t) = pF_{1,X}(t) + (1-p)F_{2,X}(t)$; *p* is the mixing parameter. This implies that the lifetime of a component is a mixture. Thus, we suppose that components arise from a mixed population of "weak" and "strong" components. The reliability characteristics of such mixtures have been extensively studied [11], although maintenance policies have been studied to a lesser degree [5],[12].

We assume that an inspection is perfect: that is, it will reveal a failure with probability 1 if the component is in the failed state; it will reveal a defect with probability 1 if the component is in the defective state; it has no effect on the lifetime of the component—the system is "as good as old" after a positive inspection in which the system is found to be good; and the component is replaced immediately following a negative inspection (in which the component is found to be either defective or failed).

The policy we describe is age-based (replacement and inspections times are based on the age of the component from new), and on replacement, future inspections and replacement are rescheduled. Although in practice a block or periodic type policy [13] is more likely to be used, the results will be very similar. Exact mathematical expressions for quantities of interest in a block type policy will be however much more difficult to obtain.

### 3. Downtime, uptime, and cost

Denote the four decision variables in our problem by $\mathbf{T} = (T_1, T_2, M_1, M_2)$. Vaurio [1] considers the (*T,M*) single phase inspection policy for a two-state single component preparedness system, where *T* is the inspection interval and *M* is the number of inspections until replacement. Jia and Christer [3] consider pure inspection for a three-state single component system, albeit with aperiodic inspections (at $t_1, t_2, \ldots$). Extending these results to two inspection phases and a final preventive replacement, the average downtime in a renewal cycle is:

$$D(\mathbf{T}) = \sum_{i=1}^{M_1} \int_{(i-1)T_1}^{iT_1} \left\{ d_{\mathrm{D}}[1 - F_H(iT_1 - x)] + \int_0^{iT_1 - x} [d_{\mathrm{F}} + iT_1 - (x+h)] f_H(h) dh \right\} f_X(x) dx$$

$$+ \sum_{i=1}^{M_2} \int_{M_1 T + (i-1)T_2}^{M_1 T_1 + iT_2} \left\{ d_{\mathrm{D}}[1 - F_H(M_1 T_1 + iT_2 - x)] + \int_0^{M_1 T_1 + iT_2 - x} [d_{\mathrm{F}} + M_1 T_1 + iT_2 - (x+h)] f_H(h) dh \right\} f_X(x) dx$$

$$\int_{M_1 T_1 + M_2 T_2}^{\infty} d_{\mathrm{G}} f_X(x) dx, \qquad (1)$$

where $d_{\mathrm{D}}$, $d_{\mathrm{F}}$ and $d_{\mathrm{G}}$ are the downtimes due to a replacement of a defective component at inspection, replacement of a failed component at inspection, and replacement of a good component at the planned replacement at $M_1T_1 + M_2T_2$, respectively. The expression (1) is obtained by conditioning on a defect arrival in each of the possible inspection intervals, $[0,T_1]$, $(T_1,2T_1]$, $\ldots,((M_1-1)T_1,M_1T_1],((M_1T_1,M_1T_1+T_2]$, $\ldots$, $(M_1T_1+(M_2-1)T_2,M_1T_1+M_2T_2]$. Then, if this defect arises at time *x,* the component does not fail if the delay time *H* is greater than the time to subsequent inspection, and does fail otherwise; these are the two terms inside the integral with respect to *x*. The final term considers the case that a defect arises beyond the age at preventive replacement, $M_1T_1 + M_2T_2$. This last term can be written more simply as $d_{\mathrm{G}}[1-F_X(M_1T_1+M_2T_2)]$. The expressions for cost and availability below arise in a similar manner.

The downtimes could be formulated slightly differently. We might for example distinguish unplanned replacements from planned replacements, so that the downtime due to replacement at inspection (in the event that a component is defective) is $d_{\mathrm{U}}$ and downtime due to replacement at the planned, preventive replacement age $M_1T_1 + M_2T_2$ is $d_{\mathrm{P}}$. Our idea here is that only the replacement at the end of the inspection phase at $M_1T_1 + M_2T_2$ is planned; all other replacements are unplanned and incur the same downtime regardless of whether a defective or failed component is replaced. In this case the downtime formulation in expression (1) becomes:

$$
\begin{aligned}
D'(\mathbf{T}) &= \sum_{i=1}^{M_1} \int_{(i-1)T_1}^{iT_1} \left\{ d_{\mathrm{U}}[1-F_H(iT_1-x)] + \int_0^{iT_1-x} [d_{\mathrm{U}} + iT_1 - (x+h)] f_H(h)dh \right\} f_X(x)dx \\
&+ \sum_{i=1}^{M_2-1} \int_{M_1T_1+(i-1)T_2}^{M_1T_1+iT_2} \left\{ d_{\mathrm{U}}[1-F_H(M_1T_1+iT_2-x)] + \int_0^{M_1T_1+iT_2-x} [d_{\mathrm{U}} + M_1T_1 + iT_2 - (x+h)] f_H(h)dh \right\} f_X(x)dx \\
&+ \int_{M_1T_1+(M_2-1)T_2}^{M_1T_1+M_2T_2} \left\{ d_{\mathrm{P}}[1-F_H(M_1T_1+M_2T_2-x)] + \int_0^{M_1T_1+M_2T_2-x} [d_{\mathrm{P}} + M_1T_1 + M_2T - (x+h)] f_H(h)dh \right\} f_X(x)dx \\
&+ d_{\mathrm{P}}[1-F_X(M_1T_1+M_2T_2)], \qquad (2)
\end{aligned}
$$

Apart from setting $d_{\mathrm{U}} = d_{\mathrm{F}} = d_{\mathrm{D}}$ and $d_{\mathrm{P}} = d_{\mathrm{G}}$, the difference between expression (2) and expression (1) above is that the final inspection interval must be treated separately as it ends with a planned replacement.

We assume in expressions (1) and (2) that the downtime due to inspection $d_{\mathrm{I}}$ is zero. Jia and Christer [3] suppose otherwise. However, there is a slight problem with their formulation in this case. This is because if $d_{\mathrm{I}} \neq 0$ then, for example, after the first inspection, time has moved on from *T* to $T + d_{\mathrm{I}}$ but this is ignored as far as component age is concerned. In their model there are two time scales operating, one for downtime, one for the component (system) age, that are synchronized initially but not subsequent to inspection. One way round this is to assume the system does not age during inspection even though the inspection time is non-zero. In this case the expression for the downtime becomes:

$$D(\mathbf{T}) = \sum_{i=1}^{M_1} \int_{(i-1)T_1}^{iT_1} \left\{ id_{\mathrm{I}} + d_{\mathrm{D}}[1 - F_H(iT_1 - x)] + \int_0^{iT_1 - x} [d_{\mathrm{F}} + iT_1 - (x+h)] f_H(h)dh \right\} f_X(x)dx$$

$$+ \sum_{i=1}^{M_2} \int_{M_1T_1+(i-1)T_2}^{M_1T_1+iT_2} \left\{ (M_1T_1 + i)d_I + d_{\mathrm{D}}[1 - F_H(M_1T_1 + iT_2 - x)] + \int_0^{M_1T_1+iT_2-x} [d_{\mathrm{F}} + M_1T_1 + iT_2 - (x+h)] f_H(h)dh \right\} \times$$

$$f_X(x)dx \quad + \{(M_1T_1 + M_2T_2)d_I + d_{\mathrm{G}}\}[1 - F_X(M_1T_1 + M_2T_2)]. \tag{3}$$

One might argue that there is in fact no downtime during an inspection, as the system is still available while "on test". We make this assumption and use (1). Note, (3) can also be modified to consider the distinction between unplanned and planned downtimes in a similar manner to (2).

When we consider the unplanned and planned replacement case, the expected cycle length can be obtained by noting the following: if a defect arises in the first inspection interval, then the cycle length is necessarily $T_1 + d_{\mathrm{U}}$; if a defect arises in the second inspection interval, then the cycle length is necessarily $2T_1 + d_{\mathrm{U}}$;…etc. On the other hand if a defect arises in the final inspection interval, or does not arise prior to the planned preventive replacement, then the cycle length is necessarily $M_1T_1 + M_2T_2 + d_{\mathrm{P}}$. Whence the expect cycle length is

$$V'(\mathbf{T}) = \sum_{i=1}^{M_1} (d_{\mathrm{U}} + iT_1)\{F_X(iT_1) - F_X((i-1)T)\}$$

$$+ \sum_{i=1}^{M_2-1} (d_{\mathrm{U}} + M_1T_1 + iT_2)\{F_X(M_1T_1 + iT_2) - F_X(M_1T_1 + (i-1)T_2)\}$$

$$+ (d_{\mathrm{P}} + M_1T_1 + M_2T_2)\{1 - F_X(M_1T_1 + (M_2 - 1)T_2)\}$$

$$= T_1 \sum_{i=0}^{M_1-1} R(iT_1) + T_2 \sum_{i=0}^{M_2-1} R(M_1T_1 + iT_2) + d_{\mathrm{P}} R\{M_1T_1 + (M_2 - 1)T_2\} + d_{\mathrm{U}} F\{M_1T_1 + (M_2 - 1)T_2\}. \tag{4}$$

Under the downtime specification with distinct downtimes for replacement of good, defective and failed components, the expected cycle length is

$$V(\mathbf{T}) = \sum_{i=1}^{M_1} \int_{(i-1)T_1}^{iT_1} \{ iT_1 + d_{\mathrm{D}}[1 - F_H(iT_1 - x)] + d_{\mathrm{F}} F_H(iT_1 - x) \} f_X(x)dx$$

$$+ \sum_{i=1}^{M_2} \int_{M_1T_1+(i-1)T_2}^{M_1T_1+iT_2} \{ M_1T_1 + iT_2 + d_{\mathrm{D}}[1 - F_H(M_1T_1 + iT_2 - x)] + d_{\mathrm{F}} F_H(M_1T_1 + iT_2 - x) \} f_X(x)dx$$

$$+ (d_{\mathrm{G}} + M_1T_1 + M_2T_2)\{1 - F_X(M_1T_1 + M_2T_2)\}. \tag{5}$$

The expected uptime per cycle is $U(\mathbf{T}) = V(\mathbf{T}) - D(\mathbf{T})$.

The expected maintenance cost per cycle is

$$C_{\mathrm{M}}(\mathbf{T}) = \sum_{i=1}^{M_1} \int_{(i-1)T_1}^{iT_1} \{c_{\mathrm{D}}[1 - F_H(iT_1 - x)] + c_{\mathrm{F}} F_H(iT_1 - x) + ic_{\mathrm{I}}\} f_X(x)dx$$

$$+ \sum_{i=1}^{M_2} \int_{M_1T_1+(i-1)T_2}^{M_1T_1+iT_2} \{c_{\mathrm{D}}[1 - F_H(M_1T_1 + iT_2 - x)] + c_{\mathrm{F}} F_H(M_1T_1 + iT_2 - x) + (M_1 + i)c_{\mathrm{I}}\} f_X(x)dx$$

$$+ \{c_{\mathrm{G}} + (M_1T_1 + M_2T_2)c_{\mathrm{I}}\}\{1 - F_X(M_1T_1 + M_2T_2)\}. \tag{6}$$

where $c_{\mathrm{G}}$, $c_{\mathrm{D}}$, and $c_{\mathrm{F}}$ are the costs of a replacement of a good, defective and failed component respectively and $c_{\mathrm{I}}$ is the cost of an inspection. Again we can reformulate the costs for planned and unplanned replacements. The cost expression becomes:

$$C'_{\mathrm{M}}(\mathbf{T}) = \sum_{i=1}^{M_1} \int_{(i-1)T_1}^{iT_1} \{c_{\mathrm{U}}[1 - F_H(iT_1 - x)] + c_{\mathrm{U}} F_H(iT_1 - x) + ic_{\mathrm{I}}\} f_X(x)dx$$

$$+ \sum_{i=1}^{M_2-1} \int_{M_1T_1+(i-1)T_2}^{M_1T_1+iT_2} \{c_{\mathrm{U}}[1 - F_H(M_1T_1 + iT_2 - x)] + c_{\mathrm{U}} F_H(M_1T_1 + iT_2 - x) + (M_1 + i)c_{\mathrm{I}}\} f_X(x)dx$$

$$+ \int_{M_1T_1+(M_2-1)T_2}^{M_1T_1+M_2T_2} \{c_{\mathrm{P}}[1 - F_H(M_1T_1 + M_2T_2 - x)] + c_{\mathrm{P}} F_H(M_1T_1 + M_2T_2 - x) + (M_1 + M_2 - 1)c_{\mathrm{I}}\} f_X(x)dx$$

$$+ \{c_{\mathrm{P}} + (M_1 + M_2 - 1)c_{\mathrm{I}}\}\{1 - F_X(M_1T_1 + M_2T_2)\}, \tag{7}$$

where $c_{\mathrm{U}}$, and $c_{\mathrm{P}}$ are the costs of unplanned and planned replacements. In this formulation (7), an inspection cost is not incurred when the system reaches the preventive, planned replacement age $M_1T_1 + M_2T_2$. Also, the cost expression (7) simplifies to

$$C'_{\mathrm{M}}(\mathbf{T}) = \sum_{i=1}^{M_1} (c_{\mathrm{U}} + ic_{\mathrm{I}})\{F_X(iT_1) - F_X((i-1)T)\}$$

$$+ \sum_{i=1}^{M_2-1} (c_{\mathrm{U}} + (M_1 + i)c_{\mathrm{I}})\{F_X(M_1T_1 + iT_2) - F_X(M_1T_1 + (i-1)T_2)\}$$

$$+ (c_{\mathrm{P}} + (M_1 + M_2 - 1)c_{\mathrm{I}})\{1 - F_X(M_1T_1 + (M_2 - 1)T_2)\}.$$

$$= c_{\mathrm{I}}\{\sum_{i=0}^{M_1-1} R(iT_1) + \sum_{i=0}^{M_2-1} R(M_1T_1 + iT_2)\} + c_{\mathrm{P}} R\{M_1T_1 + (M_2 - 1)T_2\} + c_{\mathrm{P}} F\{M_1T_1 + (M_2 - 1)T_2\}.$$

**4. Two-state failure model case**

When the time in the defective state is zero, and the component moves directly from the good to the failed state, the downtime, uptime and the cost simplify. For the case in which we distinguish downtimes due to a replacement of a defective component at inspection, and replacement of a good component at the planned replacement, they become:

$$V(\mathbf{T}) = T_1 \sum_{i=0}^{M_1-1} R(iT_1) + T_2 \sum_{i=0}^{M_2-1} R(M_1T_1 + iT_2) + d_{\mathrm{G}} R(M_1T_1 + M_2T_2) + d_{\mathrm{F}} F(M_1T_1 + M_2T_2), \tag{8}$$

$$U(\mathbf{T}) = \int_0^{M_1T_1+M_2T_2} R(t)\mathrm{d}t\,, \tag{9}$$

and

$$C_{\mathrm{M}}(\mathbf{T}) = c_{\mathrm{I}}\{\sum_{i=0}^{M_1-1} R(iT_1) + \sum_{i=0}^{M_2-1} R(iT_2 + M_1T_1)\} + c_{\mathrm{G}}R(M_1T_1 + M_2T_2) + c_{\mathrm{F}}F(M_1T_1 + M_2T_2)\,. \tag{10}$$

These were derived in Cavalcante et al. [2]. The expression (8) can be obtained from expression (5) by setting the delay time to zero (no defective state), that is by setting $F_H(h) = 1$ for all $h \ge 0$. The cost expression (10) can similarly be obtained from expression (6). Further, by setting $M = M_1 + M_2$, $T = T_1 = T_2$, the formulae above reduce to those in Vaurio [1].

When we distinguish planned from unplanned replacements, the expected cycle length is the same as expression (8), and expected maintenance cost is the same as expression (7). The difference between the two-state failure case and the three-state failure case comes in the expected uptime; for the two-state case this is as in expression (9) which is quite different from three-state case. Note, the uptime in the three-state case is not derived directly; this is because it is easier to work with the downtime. The converse is true for the two-state failure model. Furthermore, notice that in the case which distinguishes planned from unplanned replacements, if there is a transition from the good state, the maintenance cost in a cycle and the cycle length are the same regardless of whether this transition is to the defective state or the failed state. This is because the component undergoes an unplanned replacement (or planned replacement if it is the final cycle) regardless of whether it is defective or failed.

## 5. Decision criteria

Suppose now that demand for the function of the system in the event of an emergency occurs according to a Poisson process with rate $\mu$. Then the expected number of unmet demands in a cycle is $\mu D(\mathbf{T})$. If $c_{\mathrm{UD}}$ is the cost of an unmet demand, then the expected total cost per cycle is

$$C_{\mathrm{T}}(\mathbf{T}) = C_{\mathrm{M}}(\mathbf{T}) + \mu c_{\mathrm{UD}} D(\mathbf{T})\,,$$

assuming that the system is not replaced when failed on-demand. At face value this assumption is questionable. Calculation of the total cost per cycle if this assumption is relaxed is an interesting problem. However, if demand events are rare, the total cost above will provide a reasonable approximation to the exact value.

The average availability is

$$A(\mathbf{T}) = U(\mathbf{T})/V(\mathbf{T})\,, \tag{11}$$

the long-run average maintenance cost per unit time is

$$C_1(\mathbf{T}) = C_{\mathrm{M}}(\mathbf{T})/V(\mathbf{T})\,, \tag{12}$$

and the long-run average total cost per unit time is

$$C_2(\mathbf{T}) = C_{\mathrm{T}}(\mathbf{T})/V(\mathbf{T})\,. \tag{13}$$

Note that the average availability is the same as the probability that the system is up on demand. For optimization, one can either determine that minimum long-run average maintenance cost per

unit time that meets some average availability constraint, or minimize the long-run average total cost per unit time. The former decision problem avoids the difficulty associated with modelling the effect of the demand process. Vaurio [1] does not discuss this issue.

We have also assumed that the downtime due to inspections is negligible. As discussed above modification of the decision criteria to allow non-negligible inspection downtimes is also not straightforward.

## 6. Case study

The scale and extent of natural gas distribution systems are growing rapidly [14], and maintenance planning to ensure continuity of supply is an important problem. Valves are preparedness systems that are used to ensure continuity of supply in the event of a failure on the network. They are also for containment in the event of a leak [15]. Valves are subject to functionality tests or inspections. We consider the inspection policy for a single valve in this case study. Consideration of the interactions with other components in the distribution system is a potential subject for a fuller study.

Cost and downtime parameter values for a typical valve in the network were obtained subjectively from an expert. In order to maintain confidentiality, all costs are recorded relative to the cost of inspection. Thus the cost of inspection is the taken to be the unit cost. The time unit is also arbitrarily specified. Plausible values for the parameters of the mixed failure time distribution were a matter of debate. For the weak items, a small characteristic life might be explained by poor installation or might reflect the use of a different, lower quality component supplier [16]. We use Weibull distributions to model the time to defect arrival. In the base case, we set the characteristic life of weak components $\eta_1$=2 and shape parameter $\beta_1$=2.5. For the strong components, we set the characteristic life $\eta_2$=18 and shape parameter $\beta_2$=5. In this case, the lifetimes of the weak and strong are well separated in the mixture. The proportion of weak items is taken to be 0.10. For the time in the defect state, the delay time, we use an exponential distribution with mean ½ ($\lambda = 2$) in the base case. We do not consider distinct delay time distributions for weak and strong components although we might easily do so. Other distributions for the delay time might also be used.

A sensitivity of the results to the values of the model and policy parameters is carried out. We present results for the two-phase inspection interval policy $(T_1, T_2, M_1, M_2)$, and also for comparison purposes for the simple fixed interval or single phase policy ($T,M$). Two optimization criteria are used. In the first, the long-run average maintenance cost per unit time (equation 12) is minimized subject to an average availability (equation 11) requirement: min $C_1$ such that $A \geq a$. In the second, we simply minimize the long-run average total cost per unit time (equation 13) taking account of the potential cost of an unmet demand: min $C_2$. In this second problem, we set $\mu$=0.01 and $c_{\mathrm{UD}}$ =25000. Note that we strictly need only vary $\mu \times c_{\mathrm{UD}}$ in equation 1, so that a high cost of unmet demand and a low rate of occurrence of demands is equivalent to a low cost of unmet demand and a high rate of occurrence of demands. Whether the policy should be the same in each of these cases is another matter and is beyond the scope of this paper. Where the value of $\mu \times c_{\mathrm{UD}}$ is difficult to specify, it may be instructive to determine the value that leads to a sensible value for the average availability in the min $C_2$ problem.

*6.1 Results*

The behaviour of optimal policy is explored in table 1. Here the results (optimal values of decision variables, and optimal long-run costs and average availability) for the base case are shown in the first row. For the same parameter set, we then consider alternative decision criteria; in particular, we vary the availability requirement. Also, special cases of the policy are presented. We then vary the parameter values—changes to parameter values from the base case are shaded to allow clear comparisons. In table 2, we do similar but for the two-state failure model case. Figures 1-3 provide illustrations of the effect of the decision variables on the decision criteria in the base case for the three-state failure case.

It is interesting to note that the optimum policy changes markedly depending on the availability requirement. If the required availability is very high ($a$=0.997) then we observe $T_1^* < T_2^*$ (in the manner of the two-state failure case). Further, when the lifetimes of weak and strong components in the mixture become more distinct and well separated, the separation of the phases increases, i.e. $T_1$ and $T_2$ become more distinct. When the required availability is not so high (0.98-0.99), the phases are quite the reverse with $T_1^* > T_2^*$, but only when the mean delay time is large and the mixing parameter is small. Thus, if the availability requirement is not too large then the first inspection phase is not really required. However, as the availability requirement increases, then we reach a point for which $T_1^* < T_2^*$ . When minimising the long run average total cost per unit time (min $C_2$) a similar effect would be observed when varying the average cost of unmet demand, $\mu \times c_{\text{D}}$. This is because min $C_2$ and min $C_1$ s.t. $A \geq a$ are broadly equivalent for particular values of $a$ and $\mu \times c_{\text{D}}$.

In rows 13-15 of table 1, optimum values for policies that are special cases (age based replacement, $M_1 = 1$, $M_2 = 0$; single phase pure inspection, $M_1 = \infty$; two-phase pure inspection, $M_1 < \infty$, $M_2 = \infty$) are presented. These policies are significantly more expensive. Thus even with the existence of a defective state, pure inspection with a reasonable inspection frequency is not sufficient to guarantee high availability. However, with regard to the single phase inspection and replacement policy ($M,T$), costs and availability (row 12, table 1) are closer to the those of the two phase inspection and replacement policy. For both the three-state failure case and the two-state failure case, the two-phase policy appears to produce cost savings over the single phase policy of the order of 5% over a broad range of parameter values.

In other respects, the decision variables appear to act as expected. Generally, variations in replacement and inspection costs, and downtimes, have a much smaller effect on optimum policies than variations in the failure distribution parameters, and here variation in the mixing parameter appears to have the largest effect.

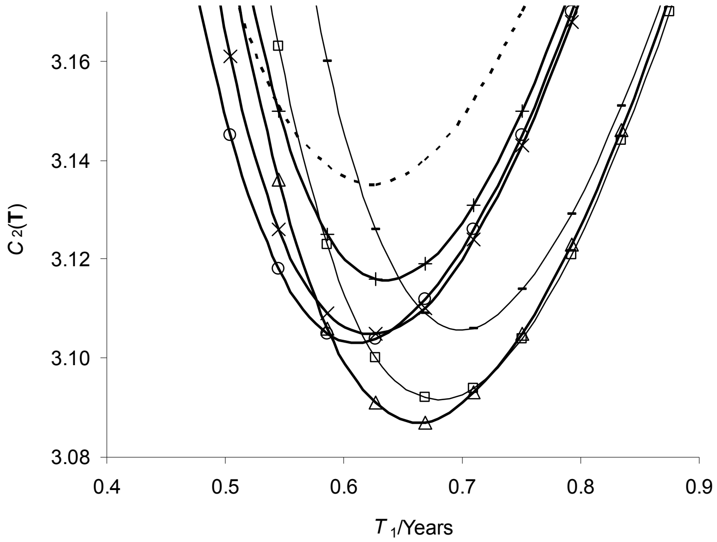


Figure 1. Long-run average total cost per unit time, $C_2(\mathbf{T}) = C_T(\mathbf{T})/V(\mathbf{T})$, as a function of $T_1$ for $M_1$=5 and $M_2$=5 (—‾—), $M_1$=5 and $M_2$=6 (—□—), $M_1$=5 and $M_2$=7 (—Δ—), $M_1$=6 and $M_2$=5 (—+—), $M_1$=6 and $M_2$=6 (—X—), $M_1$=6 and $M_2$=7(—o—), $M_1$=7 and $M_2$=6 (.....). $T_2$ held at the optimum value specific for each pair ($M_1$,$M_2$). Parameter values: $p$=0.1, $\eta_1$=2, $\eta_2$=18, $\beta_1$=2.5, $\beta_2$=5, $c_G$=12, $c_F$ =18, $c_I$=1, $\mu$=0.01, $c_D$=25000, $d_D$=0.0025, $d_F$=0.005, $d_G$=0.001 (base case).

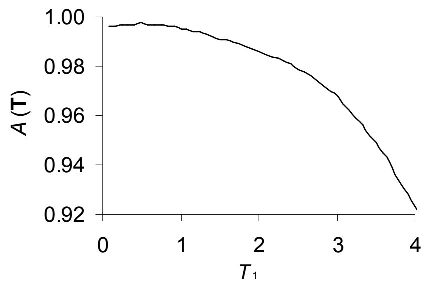


Figure 2. Average availability, $A(\mathbf{T}) = U(\mathbf{T})/V(\mathbf{T})$, as a function of inspection interval, $T_1$ . Parameter values as figure 1 (base case). $T_2$, $M_1$ and $M_2$ held at global optimum values

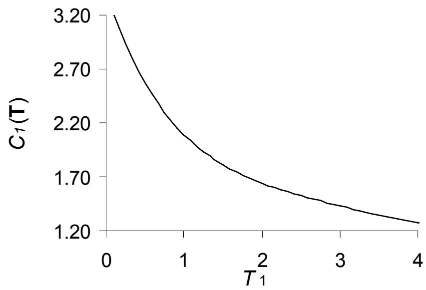


Figure 3. Long-run average maintenance cost per unit time, $C_1(\mathbf{T}) = C_M(\mathbf{T})/V(\mathbf{T})$, as a function of inspection interval, $T_1$. Parameter values as figure 1 (base case). $T_2$, $M_1$ and $M_2$ held at global optimum values.

*6. 2 Discussion of the results*

In the three-state case, it appears that the defective state provides an opportunity to anticipate failure. This effect is reflected in the values of the availability, maintenance cost and total cost criteria. The greater the mean delay time, the bigger are these advantages when compared with the two-state case. The most expensive particular policy, for the three-state case is age based replacement. But, even in this case the economy accruing due to the existence of a defective state is about 25% when compared with the two-state case. This may to an extent be due to the fact that the mean lifetime in the three-state case is slightly longer than in the two-state case, as the mean lifetime is the sum of the mean time to defect arrival and mean delay time.

An interesting observation is in regard to the fact that for small values of $\lambda$ (large mean delay time), the two-phase policy does not necessarily mimic a burn-in process, where the initial inspection phase aims to prevent failures due to the possibility of the presence of a weak component. It depends on the decision criterion. If the availability requirement is not too large, then we observe $T_1^* > T_2^*$. Thus, it appears that the defective state provides a warning against early failure even if a weak component is in situ, and a conservative first inspection phase is not required. Only when $\lambda$ is large (small mean delay time), or the availability requirement is large (close to 100%), do we require a conservative first inspection phase (small $T_1^*$). There will be a critical value for the mean delay time at which one would switch from a conservative to liberal first inspection phase. Of course this critical value will depend on the size of *p*, and other parameters. If *p*=0, then we would expect $T_1^* > T_2^*$. This then suggests that if the component is instrumented (so that any defect present is observable), then early inspections will not be necessary. In this way one might quantify the benefit of monitoring (continuously or periodically) the system state.

Furthermore, our analysis of the two problems, 1) minimizing the long-run average maintenance cost subject to an availability constraint, and 2) minimizing the long-run average total cost, indicate that setting the value for the required availability and specifying the average cost of unmet demand are equivalent. For example, we can see from table 1 that $\mu \times c_{\mathrm{D}}$=250 is broadly equivalent to *a*=0.997 (comparing results in rows 8 and 10) and that $\mu \times c_{\mathrm{D}}$=100 is broadly equivalent to *a*=0.995 (comparing results in row 7 in table 1 with results in row 7 from the foot of table 1 continued). A similar conclusion can be drawn for the two-state failure model case. This analysis can facilitate the determination of such policy inputs. For example, one might ask engineers to independently propose values for the required availability and the cost of unmet demand, and then establish whether these inputs are broadly comparable in the magnitude of their effect on the inspection policy. If not, then the input values might be adjusted accordingly.

When $\lambda \to \infty$ (no defective state), then results from the two state model are re-captured. This is some way provides a check on the results for the three-state failure case.

The results we present are intended to be indicative of the behaviour of optimum policy. The effect of the use of unreliable components in the network, and thus the benefit of carrying out more thorough replacements or using higher quality suppliers, may be investigated in the manner we indicate, and likewise for the dis-benefits of the use of maintenance polices which are simpler to manage. This approach then can support decision making not just about optimal timing of inspections but also about broader issues to do with the efficacy of maintenance management.

In some sense we could say that the problem we consider is a multi-criteria one (involving cost and availability), although we are not applying a multi-criteria method. Such a method would require an analysis of the trade-off between multiple criteria, based on a model of the preferences of a decision maker (see [17]-[20]). This decision maker would be a manager involved in the process and taking responsibility for the consequences of unavailability and increased costs. In practice this may not be necessary, depending of the context and characteristics of the system analysed. The manager may merely wish to use the insights provided in this paper to guide decision making. However, a multi criteria approach will be useful in situations where a decision maker wishes to explore in detail the trade-off between cost and availability.

## 7. Concluding remarks

In this paper we propose a two-phase inspection policy for a single component preparedness system, in which the component itself may arise from a heterogeneous population and the component may be in one of three states: good, defective or failed. Criteria for the determination of optimal policy are presented. The work extends that of Vaurio [1] who considered single phase policies for components from a homogeneous population. It also extends that of Cavalcante et al. [2] who consider the two-state case in which a component can only be good or failed. Reliability studies of mixed failure distributions are well documented. Maintenance policy modelling when components in systems have mixed failure distributions is less well developed, and this paper makes a contribution to this area. We also address a point made by Vaurio [1] who argues that there has been little attention on the economics of systems with dormant failures. Our policy has the property that it not only determines when to carry out maintenance but it also it helps the decision maker to determine what type of maintenance to carry out.

We consider two optimization problems: minimization of the long-run maintenance cost per unit time subject to a required average availability; and, more simply, minimization of the long-run total cost per unit time (that includes the cost of potential, unmet demand). We indicate that these two problems are broadly equivalent given particular values of the cost of an unmet demand event and the required average availability.

When the underlying distributions in the mixture that model the heterogeneous population of components are well separated, then maintenance cost savings are obtainable. Variation in the mixture distribution parameters tends to have a greater effect on optimum policy than inspection and replacement costs and downtimes.

The existence of a defective state has an interesting effect. Broadly we find that if the mean delay time is large the two-phase inspection policy does not mimic a burn-in process, where the initial inspection phase has the function of mitigating against failures due to the presence of weak components. This effect was not anticipated by us. Instead, the defective state provides sufficient warning against early failure in order to meet a reasonable level of availability. When the mean delay time is small or if a very high availability is required, then the first inspection phase becomes more conservative ($T_1^* < T_2^*$). The existence of a defective state also has significant effect on costs. These effects suggest to us that one could quantify the benefit of monitoring (continuously or periodically) the system state using the model we propose. We also find that in spite of the

existence of a defective state, a pure inspection policy is sub-optimal with respect to an inspect and replace policy; this is a conclusion that we had not anticipated.

In the problems we consider, we assume that the functional state of the system is established only by a test. In practice demand events would, typically, also indicate the state of the system. When considering the long run total cost, including the cost of potential unmet demands, we do not consider this latter possibility in our model. If demands are rare events, then the approach here will provide a reasonable approximation. If demand events are relatively more frequent, then it would be interesting to consider a model that allows for replacements to occur immediately following unmet demand events. In this way, we would have to consider the superposition of the demand process and the failure process when determining expressions for the decision criteria. This would be an interesting topic for further research.

Further questions remain. The work could be conducted using multi-criteria methods in order to deal with availability in a different way. One might try to modify the expressions for the quantities of interest when downtime due to inspection is non-zero. Also, one might consider distinct delay time distributions for weak and strong components. Finally, a multi-component extension would also be of further interest.

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Table 1. For the three-state failure model case, optimal values of decision variables, long run costs per unit time, and average availability, $A$, for various values of the parameters of the failure model and inspection policy. Unit of time is unspecified for reasons of confidentiality. Results from three policies reported: minimization of maintenance cost subject to required availability (min $C_1$ *s.t.* $a \geq 0.99$); minimization of total cost, including cost of unmet demand (min $C_2$); minimization of total cost for a single phase policy, $M=M_1=M_2$, $T=T_1=T_2$, (min $C_2$ $(M,T)$).

| mixed failure distribution parameters | | | | | | cost parameters | | | | | downtimes | | | optimum values of decision variables, long run cost (*Cost*), and average availability | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\beta_1$ | $\eta_1$ | $\beta_2$ | $\eta_2$ | $p$ | $\lambda$ | $c_I$ | $c_G$ | $c_F$ | $M$ | $c_D$ | $d_F$ | $d_D$ | $d_G$ | | $M_1$ | $M_2$ | $T_1$ | $T_2$ | *Cost* | $A$ |
| 2.5 | 2 | 5 | 18 | .10 | 2 | 1 | 12 | 18 | .01 | 25000 | .005 | .0025 | 0.001 | min $C_1$ *s.t.* $A$=0.99 | 5 | 3 | 2.04 | 1.10 | 1.67 | 99.0 |
| | | | | | | | | | | | | | | min $C_1$ *s.t.* $A$=0.99 $(M,T)$ | 7 | | 1.78 | | 1.71 | 99.0 |
| | | | | | | | | | | | | | | min $C_1$ *s.t.* $A$=0.98 | 3 | 2 | 3.30 | 1.99 | 1.43 | 98.0 |
| | | | | | | | | | | | | | | min $C_1$ *s.t.* $A$=0.98 $(M,T)$ | 5 | | 2.74 | | 1.45 | 98.0 |
| | | | | | | | | | | | | | | min $C_1$ *s.t.* $A$=0.97 | 1 | 1 | 3.18 | 9.07 | 1.32 | 97.0 |
| | | | | | | | | | | | | | | min $C_1$ *s.t.* $A$=0.97 $(M,T)$ | 4 | | 3.54 | | 1.34 | 97.0 |
| | | | | | | | | | | | | | | min $C_1$ *s.t.* $A$=0.995 | 4 | 5 | 0.90 | 1.54 | 2.03 | 99.5 |
| | | | | | | | | | | | | | | min $C_1$ *s.t.* $A$=0.997 | 5 | 7 | 0.68 | 1.10 | 2.34 | 99.7 |
| | | | | | | | | | | | | | | min $C_1$ *s.t.* $A$=0.997 $(M,T)$ | 14 | | 0.83 | | 2.40 | 99.7 |
| | | | | | | | | | | | | | | min $C_2$ | 5 | 7 | 0.70 | 1.08 | 3.09 | 99.7 |
| | | | | | | | | | | | | | | min $C_1$ *s.t.* $A$=0.994 | 3 | 5 | 1.07 | 1.64 | 1.93 | 99.4 |
| | | | | | | | | | | | | | | min $C_2$ $(M,T)$ | 14 | | 0.83 | | 3.15 | 99.7 |
| | | | | | | | | | | | | | | min $C_2$ | 1 | 0 | 1.92 | | 9.65 | 98.9 |
| | | | | | | | | | | | | | | min $C_2$ | $\infty$ | | 0.49 | | 4.06 | 99.5 |
| | | | | | | | | | | | | | | min $C_2$ | 1 | $\infty$ | 1.18 | 0.49 | 4.00 | 99.6 |
| 2.5 | 2 | 5 | 18 | .10 | 1 | 1 | 12 | 18 | .01 | 25000 | .005 | .0025 | 0.001 | min $C_1$ *s.t.* $A$=0.99 | 4 | 3 | 2.61 | 1.33 | 1.49 | 99.0 |
| | | | | | | | | | | | | | | min $C_2$ | 4 | 7 | 0.81 | 1.17 | 2.75 | 99.8 |
| | | | | | | | | | | | | | | min $C_2$ $(M,T)$ | 12 | | 0.99 | | 2.79 | 99.7 |
| 2.5 | 2 | 5 | 18 | .10 | 6 | 1 | 12 | 18 | .01 | 25000 | .005 | .0025 | 0.001 | min $C_1$ *s.t.* $A$=0.99 | 3 | 4 | 1.11 | 1.98 | 1.89 | 99.0 |
| | | | | | | | | | | | | | | min $C_2$ | 6 | 7 | 0.50 | 1.00 | 3.57 | 99.6 |
| | | | | | | | | | | | | | | min $C_2$ $(M,T)$ | 15 | | 0.73 | | 3.68 | 99.6 |
| 2.5 | 2 | 5 | 18 | .10 | 12 | 1 | 12 | 18 | .01 | 25000 | .005 | .0025 | 0.001 | min $C_1$ *s.t.* $A$=0.99 | 3 | 4 | 1.07 | 1.89 | 1.97 | 99.0 |
| | | | | | | | | | | | | | | min $C_2$ | 6 | 6 | 0.50 | 1.10 | 3.79 | 99.5 |
| | | | | | | | | | | | | | | min $C_2$ $(M,T)$ | 15 | | 0.71 | | 3.93 | 99.5 |

Table 1 continued

| mixed failure distribution parameters | | | | | cost parameters | | | | | | downtimes | | | optimum values of decision variables, long run cost (*Cost*), and average availability | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\beta_1$ | $\eta_1$ | $\beta_2$ | $\eta_2$ | $p$ | $\lambda$ | $c_I$ | $c_G$ | $c_F$ | $\mu$ | $c_D$ | $d_F$ | $d_D$ | $d_G$ | | $M_1$ | $M_2$ | $T_1$ | $T_2$ | *Cost* | $A$ |
| 2.5 | 2 | 5 | 18 | .05 | 2 | 1 | 12 | 18 | .01 | 25000 | .005 | .0025 | 0.001 | min $C_1$ *s.t.* $A$=0.99 | 4 | 2 | 2.66 | 1.30 | 1.49 | 99.0 |
| | | | | | | | | | | | | | | min $C_2$ | 7 | 4 | 1.20 | 0.75 | 2.80 | 99.7 |
| 2.5 | 2 | 5 | 18 | .15 | 2 | 1 | 12 | 18 | .01 | 25000 | .005 | .0025 | 0.001 | min $C_1$ *s.t.* $A$=0.99 | 3 | 5 | 1.13 | 1.87 | 1.83 | 99.0 |
| | | | | | | | | | | | | | | min $C_2$ | 6 | 8 | 0.60 | 1.00 | 3.31 | 99.7 |
| 1.5 | 2 | 5 | 18 | .10 | 2 | 1 | 12 | 18 | .01 | 25000 | .005 | .0025 | 0.001 | min $C_1$ *s.t.* $A$=0.99 | 6 | 3 | 1.82 | 1.03 | 1.69 | 99.0 |
| | | | | | | | | | | | | | | min $C_2$ | 5 | 8 | 0.70 | 0.97 | 3.12 | 99.7 |
| 3.5 | 2 | 5 | 18 | .10 | 2 | 1 | 12 | 18 | .01 | 25000 | .005 | .0025 | 0.001 | min $C_1$ *s.t.* $A$=0.99 | 4 | 3 | 2.46 | 1.11 | 1.63 | 99.0 |
| | | | | | | | | | | | | | | min $C_2$ | 5 | 6 | 0.60 | 1.22 | 3.07 | 99.7 |
| 2.5 | 2 | 5 | 18 | .10 | 2 | 1 | 8 | 18 | .01 | 25000 | .005 | .0025 | 0.001 | min $C_1$ *s.t.* $A$=0.99 | 2 | 3 | 1.59 | 2.62 | 1.37 | 99.0 |
| | | | | | | | | | | | | | | min $C_2$ | 5 | 5 | 0.68 | 1.28 | 2.74 | 99.7 |
| 2.5 | 2 | 5 | 18 | .10 | 2 | 1 | 16 | 18 | .01 | 25000 | .005 | .0025 | 0.001 | min $C_1$ *s.t.* $A$=0.99 | 6 | 4 | 1.84 | 0.95 | 1.92 | 99.0 |
| | | | | | | | | | | | | | | min $C_2$ | 5 | 9 | 0.70 | 0.95 | 3.44 | 99.7 |
| 2.5 | 2 | 5 | 18 | .10 | 2 | 1 | 12 | 12 | .01 | 25000 | .005 | .0025 | 0.001 | min $C_1$ *s.t.* $A$=0.99 | 5 | 4 | 2.01 | 1.03 | 1.58 | 99.0 |
| | | | | | | | | | | | | | | min $C_2$ | 5 | 7 | 0.67 | 1.08 | 3.08 | 99.7 |
| 2.5 | 2 | 5 | 18 | .10 | 2 | 1 | 12 | 24 | .01 | 25000 | .005 | .0025 | 0.001 | min $C_1$ *s.t.* $A$=0.99 | 5 | 2 | 2.06 | 1.19 | 1.77 | 99.0 |
| | | | | | | | | | | | | | | min $C_2$ | 5 | 7 | 0.70 | 1.07 | 3.14 | 99.7 |
| 2.5 | 2 | 5 | 18 | .10 | 2 | 1 | 12 | 18 | .01 | 10000 | .005 | .0025 | 0.001 | min $C_1$ *s.t.* $A$=0.99 | 5 | 3 | 2.04 | 1.10 | 1.67 | 99.0 |
| | | | | | | | | | | | | | | min $C_2$ | 3 | 6 | 1.00 | 1.44 | 2.52 | 99.5 |
| 2.5 | 2 | 5 | 18 | .10 | 2 | 1 | 12 | 18 | .01 | 25000 | .003 | .0025 | | min $C_1$ *s.t.* $A$=0.99 | 5 | 3 | 2.04 | 1.10 | 1.67 | 99.0 |
| | | | | | | | | | | | | | | min $C_2$ | 5 | 7 | 0.66 | 1.08 | 3.08 | 99.7 |
| 2.5 | 2 | 5 | 18 | .10 | 2 | 1 | 12 | 18 | .01 | 25000 | .010 | .0025 | | min $C_1$ *s.t.* $A$=0.99 | 5 | 3 | 2.03 | 1.10 | 1.68 | 99.0 |
| | | | | | | | | | | | | | | min $C_2$ | 5 | 7 | 0.66 | 1.08 | 3.09 | 99.7 |
| 2.5 | 2 | 5 | 18 | .10 | 2 | 1 | 12 | 18 | .01 | 25000 | .005 | .002 | | min $C_1$ *s.t.* $A$=0.99 | 5 | 3 | 2.04 | 1.10 | 1.67 | 99.0 |
| | | | | | | | | | | | | | | min $C_2$ | 5 | 7 | 0.66 | 1.08 | 3.09 | 99.7 |

Table 2. For the two-state failure model case, optimal values of decision variables, long run costs per unit time, and average availability, $A$, for various values of the parameters of the failure model and inspection policy. Unit of time is unspecified for reasons of confidentiality. Results from three policies reported: minimization of maintenance cost subject to required availability (min $C_1$ *s.t.* $a \geq 0.99$); minimization of total cost, including cost of unmet demand (min $C_2$); minimization of total cost for a single phase policy with $M=M_1=M_2$, $T=T_1=T_2$, (min $C_2$ $(M,T)$).

| mixed failure distribution parameters | | | | | cost parameters | | | | | downtimes | | optimum values of decision variables, long run cost (*Cost*), and average availability | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\beta_1$ | $\eta_1$ | $\beta_2$ | $\eta_2$ | $p$ | $c_I$ | $c_D$ | $c_F$ | $\mu$ | $c_{UD}$ | $d_F$ | $d_D$ | | $M_1$ | $M_2$ | $T_1$ | $T_2$ | *Cost* | $A$ |
| 2.5 | 2 | 5 | 18 | .10 | 1 | 12 | 18 | .01 | 25000 | .005 | .001 | min $C_1$ *s.t.* $A$=0.99 | 4 | 4 | 0.87 | 1.81 | 2.077 | 99.0 |
| | | | | | | | | | | | | min $C_2$ | 6 | 6 | 0.54 | 1.09 | 4.092 | 99.4 |
| | | | | | | | | | | | | min $C_1$ *s.t.* $A$=0.994 | 6 | 5 | 0.56 | 1.26 | 2.602 | 99.4 |
| | | | | | | | | | | | | min $C_2$ $(M,T)$ | 15 | | 0.69 | | 4.244 | 99.4 |
| | | | | | | | | | | | | min $C_2$ | 1 | 0 | 1.75 | | 13.056 | 98.2 |
| | | | | | | | | | | | | min $C_2$ | ∞ | | 0.35 | | 6.992 | 98.8 |
| | | | | | | | | | | | | min $C_2$ | 1 | ∞ | 1.09 | 0.34 | 6.906 | 98.8 |
| 2.5 | 1 | 5 | 18 | .10 | 1 | 12 | 18 | .01 | 25000 | .005 | .001 | min $C_1$ *s.t.* $A$=0.99 | 4 | 4 | 0.64 | 2.80 | 1.974 | 99.0 |
| | | | | | | | | | | | | min $C_2$ | 5 | 5 | 0.37 | 1.47 | 3.854 | 99.5 |
| | | | | | | | | | | | | min $C_2$ $(M,T)$ | 14 | | 0.74 | | 4.264 | 99.4 |
| 2.5 | 3 | 5 | 18 | .10 | 1 | 12 | 18 | .01 | 25000 | .005 | .001 | min $C_1$ *s.t.* $A$=0.99 | 4 | 4 | 1.10 | 1.51 | 2.116 | 99.0 |
| | | | | | | | | | | | | min $C_2$ | 7 | 6 | 0.64 | 0.93 | 4.176 | 99.4 |
| | | | | | | | | | | | | min $C_2$ $(M,T)$ | 14 | | 0.74 | | 4.216 | 99.4 |
| 2.5 | 2 | 5 | 18 | .05 | 1 | 12 | 18 | .01 | 25000 | .005 | .001 | min $C_1$ *s.t.* $A$=0.99 | 2 | 2 | 1.33 | 3.36 | 1.815 | 99.0 |
| | | | | | | | | | | | | min $C_2$ | 4 | 4 | 0.80 | 1.42 | 3.519 | 99.5 |
| 2.5 | 2 | 5 | 18 | .15 | 1 | 12 | 18 | .01 | 25000 | .005 | .001 | min $C_1$ *s.t.* $A$=0.99 | 5 | 5 | 0.67 | 1.46 | 2.389 | 99.0 |
| | | | | | | | | | | | | min $C_2$ | 8 | 7 | 0.42 | 0.98 | 4.574 | 99.4 |
| 1.5 | 2 | 5 | 18 | .10 | 1 | 12 | 18 | .01 | 25000 | .005 | .001 | min $C_1$ *s.t.* $A$=0.99 | 4 | 4 | 0.93 | 1.69 | 2.119 | 99.0 |
| | | | | | | | | | | | | min $C_2$ | 7 | 7 | 0.51 | 0.93 | 4.168 | 99.5 |
| 3.5 | 2 | 5 | 18 | .10 | 1 | 12 | 18 | .01 | 25000 | .005 | .001 | min $C_1$ *s.t.* $A$=0.99 | 4 | 3 | 0.70 | 2.46 | 2.087 | 99.0 |
| | | | | | | | | | | | | min $C_2$ | 6 | 5 | 0.51 | 1.28 | 4.041 | 99.4 |
| 2.5 | 2 | 5 | 18 | .10 | 1 | 8 | 18 | .01 | 25000 | .005 | .001 | min $C_1$ *s.t.* $A$=0.99 | 4 | 4 | 0.88 | 1.81 | 1.737 | 99.0 |
| | | | | | | | | | | | | min $C_2$ | 6 | 5 | 0.55 | 1.19 | 3.699 | 99.4 |
| 2.5 | 2 | 5 | 18 | .10 | 1 | 16 | 18 | .01 | 25000 | .005 | .001 | min $C_1$ *s.t.* $A$=0.99 | 4 | 4 | 0.87 | 1.82 | 2.417 | 99.0 |
| | | | | | | | | | | | | min $C_2$ | 6 | 6 | 0.55 | 1.12 | 4.471 | 99.4 |
| 2.5 | 2 | 5 | 18 | .10 | 1 | 12 | 12 | .01 | 25000 | .005 | .001 | min $C_1$ *s.t.* $A$=0.99 | 4 | 4 | 0.87 | 1.82 | 1.976 | 99.0 |
| | | | | | | | | | | | | min $C_2$ | 6 | 6 | 0.54 | 1.1 | 3.997 | 99.4 |
| 2.5 | 2 | 5 | 18 | .10 | 1 | 12 | 24 | .01 | 25000 | .005 | .001 | min $C_1$ *s.t.* $A$=0.99 | 4 | 4 | 0.87 | 1.81 | 2.178 | 99.0 |
| | | | | | | | | | | | | min $C_2$ | 6 | 6 | 0.54 | 1.08 | 4.187 | 99.4 |