Modelling quality in replacement and inspection maintenance

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ABSTRACT: Heterogeneity in component and maintenance quality is considered in the context of age-based inspection and replacement for a single component system. A three-state component failure model is assumed, with a defective state preceding the failed state. Heterogeneity in maintenance intervention is modelled by supposing that inspections may induce the detective state. Component heterogeneity is modelled by supposing that the population of components comprises a mixture of the weak and strong, and that the mixing proportion and cost of components varies between suppliers. Within this framework, the impact, on system reliability and cost, of switching component or maintenance supplier is determined. Broadly, our finding is that one is much more inclined to switch supplier in order to obtain higher quality maintenance than to obtain higher quality components. A gas pressure control valve is used to illustrate our ideas.

KEYWORDS: Maintenance; inspection; replacement; quality; reliability; mixtures.

1. Introduction

Consider a single component system in which a component is placed in a socket and together these provide an operational function. Suppose further that the component is the subject of regular change-outs or replacements, and that the component is inspected to mitigate against early failure. For example, a traction motor is placed in a train unit and provides motive power to an axle. Traction motors are regularly changed-out: removed, overhauled or scrapped, and replaced with another motor (Chan, 2001). The linkage mechanism of a pressure regulating valve in a gas distribution network may be scheduled for annual inspections and triennial replacements (every three years) (Antaki, 2003). In this context it is natural for the system operator to be concerned with the quality of components and the quality of maintenance intervention. Under what circumstances is it economic to switch to a new component supplier? What are the operational reliability implications of switching? What are the implications of introducing components that can be condition monitored? When a highly technical system is maintained by the original equipment manufacturer (OEM), what are the implications of using a local supplier of components and maintenance? For example, the implications may be large for modern medical equipment located in a hospital in a developing country (De Vivo et al, 2004; Da Rocha et al, 2005).

Quality of maintenance can be considered in a number of ways, and our framework facilitates such consideration. The aspects of maintenance quality that we consider are novel, have important practical implications, and can be implemented in a reasonably straightforward manner. Of course, the models are only approximations to reality and it is accepted that no model can cover all aspects (Aven and Jensen, 1999, p.11). However they can still be useful for guiding maintenance decision making.

The basis of our framework is a single component system subject to inspection and preventive replacement. We assume a three stage failure process in which a component may be in one of three states: good, defective; and failed. The system is operational if the component is either good or defective, and not operational if the component is failed. Times in the good and defective states are random variables. We suppose that the system is subject to scheduled inspections and replacement. The purpose of inspection is to determine whether a component is in the good or defective state; defective components are replaced. We do not require that inspections are carried out over the entire lifetime of a component and allow that maintenance is carried out in two phases: an early inspection phase that mimics the careful application of maintenance to new equipment; and a later wear-out phase in which older equipment is subject to scheduled replacement.

On replacement, a "new" component is selected from a stockpile of components. This stockpile is a notional population of components. We suppose that this population is a mixture of two sub-populations: a weak sub-population and a strong sub-population. Components in the weak sub-population have a mean lifetime that is short relative to components from the strong sub-population. The mixing parameter is then used to represent component supplier quality, with a higher quality supplier with higher costs producing components with a smaller proportion of weak components. We might equivalently suppose that the mixing parameter represents the probability of poor installation during a preventive replacement. The operator may then wish to choose between different installers or maintainers. In this sense, the quality of a component and the quality of preventive replacement are equivalent. This is one model of quality that we propose.

In a second model, inspections are allowed to be imperfect. Inspections may induce defects. The probability that a defect is induced at inspection is used to represent the quality of a maintainer. Again higher quality maintainers are less likely to induce defects. With this model, the economic and reliability implications for inspection and the implications of switching maintenance supplier can be determined. Generally speaking, if the defect induction probability is not less than very small, there are serious implications for inspection. Also serious is the reporting of a false negative at inspection, when a component is reported as good and no action taken when in fact it is defective. Again the probability of a false negative can represent quality of maintenance. False positives, in which a component is reported as defective and replaced when it was in fact good, may be less serious (from a reliability and safety point of view) but still have economic consequences. However, for brevity and clarity, we do not consider false positives and negatives in this paper. We leave this for future work.

The mixing parameter in the mixed distribution of time in the good state can be simply interpreted as the proportion defective in the classic quality control problem (e.g. Schilling, 1982). Modern quality control methods which focus on design (e.g Logothetis and Wynn, 1989) have reduced rates of production of defective items to very low values. However, there is evidence that the problem of early life failures is extant. For example, Braasch et al. (2008) present the empirical time to failure distribution of certain telecoms devices used in automobiles. This empirical distribution is consistent with a mixed distribution, and hence the idea that at a proportion of these items are weak and fail early. Scarf et al. (2009) suggest that observed traction motor bearing failures correspond to early life failures and that wear-out failures are not generally observed as traction motors are replaced prior to wear-out. Denning (2009) suggests that as many as 10% of tank engines fail very soon after replacement. The reasons for this are complex although broadly speaking, the quality of maintenance is the predominant issue.

Quality and maintenance have been considered by others in areas that are related to our own. These are: i) the impact of the maintenance on the quality of a production process (e.g. Jonsson 2005; Kuo, 2006; Panagiotidou and Tagaras, 2007; Colledani and Tolio, 2009); ii) the modelling of imperfect maintenance through the concepts of virtual age reduction (Jack, 1998), induced failure (Christer at al., 1997; Baker 2000), and ineffective inspection (e.g. Dagg and Newby, 1998). Our approach here is different to the first of these in that we do not model the quality of output of the system directly. The second area is more related, although our model is simpler than those developed by others, can be interpreted more in terms of quality of maintenance, and can therefore, in our opinion, provide greater insight for maintainers.

In section 4, we describe fully our models of maintenance quality and their possible interpretations. This section is preceded by one in which the component and system failure model is specified (section 2) and by one in which the maintenance policy is specified (section 3). Decision criteria (overall cost, and system reliability) are introduced in section 5, and the methodology for determining these criteria follows in section 6. We then present some indicative results in the context of a pressure control valve, and conclude with a discussion of the insights for maintenance quality that our analysis provides.

2. Component and system failure model

The single component system comprises a component and socket which together perform an operational function (Ascher and Feingold, 1984). The component can be in one of three states: good, defective or failed. The time in the good state (time to defect or fault arrival) is assumed to arise from a mixture distribution: $F(t) = pF_1(t) + (1-p)F_2(t)$, where p is the mixing parameter. Thus, we suppose that components arise from a mixed population of "weak" and "strong" components, and that parameters of the sub-distributions reflect this separation. Equivalently the mixing parameter p may be interpreted as the probability of poor installation, with the weak components corresponding to poorly installed components and the strong to well installed components. Time in the defective state, the so called delay time (Christer and Redmond, 1991), is also a random variable, independent of the time to defect arrival. The time to failure or lifetime of a component is the sum of the time to defect arrival and the delay time. The system fails if and only if the component fails. Failures are immediately revealed and corrective action is required.

The reliability characteristics of components with lifetimes that are mixtures have been extensively studied (Jiang and Murthy, 1998), although maintenance policies have been studied to a much lesser degree (Scarf and Cavalcante, 2009).

3. Maintenance policy

The maintenance policy we consider is general in that it considers both inspection and preventive replacement. The policy allows us to consider quality relating to both the component supplier and the inspection process. A pure inspection policy and the age based replacement policy (Barlow and Proschan, 1965) are special cases.

The policy is as follows. When the existing component in the socket reaches age T, it is replaced by a new component. When the component reaches age $i\Delta$, $i=1,\ldots,K$, $K\Delta<T$, the component is inspected and if a defect is found the component is replaced by a new component. On component replacement the system is as good as new (renewal). T, K and Δ are the decision

variables under the control of the operator (or maintainer). This hybrid policy was proposed by Scarf et al. (2009) as a means to mitigate against the possibility of an early failure of a weak or poorly installed component. It is the operational equivalent of a burn-in maintenance policy (Jiang and Jardine, 2007). The purpose of inspection is to determine if a component is in the defective state, and it is thus the inspection that performs the role of an operational burn-in period. We would expect early replacement to be an uneconomic alternative. The policy also mimics circumstances in which one takes greater care of a system when new than when old.

One may attempt to improve component quality rather than perform early inspections or burn-in. Thus, it would seem to be a natural step forward to consider supplier and maintenance quality in the context of this policy and the mixed distribution model of failure. Other more complex policies (multi-component, minimal repair type policies, e.g. Block et al., 1998; Beichelt, 1993) would appear to be more realistic. Clearer insights on the other hand may be gained from a simpler model.

4. Supplier and maintenance quality

4.1. *A two supplier model of component quality* (*model* 1)

In this first model, the quality of a supplier is formulated in the following rather simple way. With supplier A, the preventive replacement cost is C_A and the proportion of weak components is p_A. With supplier B, the preventive replacement cost is C_A+C_B ($C_B>0$) and the proportion of weak components is p_B ($p_A>p_B$). Thus, supplier B provides components that are less likely to fail early, but charges a premium (C_B) for these. This additional cost is factored into the cost criterion described in the next section. This approach generalizes to more than 2 suppliers. The quality of a component and the quality of installation (replacement work) are mathematically equivalent in our model, and so this two supplier model can handle variation in installation quality and cost.

4.2. *A model of defect induction at inspection* (*model* 2)

Consider the three state failure process described in section 2, so that a component may be good, defective or failed. Suppose now that at an inspection of a component, a defect is induced with some probability q. With probability (1-q) the component is unaffected. That is, following an inspection at which a component is found to be in the good state (a null inspection), with probability q the component immediately enters the defective state, and with probability (1-q) the component remains in the good state with a residual time in the good state that is the same as immediately prior to the null inspection. We assume only one failure mode for the component and only one corresponding defect, and so the delay time for defects induced at inspection and for defects that arise in normal-use have the same distribution. An implication of our assumption is that if a defect is induced at inspection, a normal-use defect cannot subsequently arise (in the same component). We call this model 2. This model can be used to represent quality of inspection, and immediate questions of interest are as follows. For what values of q is it uneconomic to perform inspections? What are the operational reliability implications for $q>0$? These questions are considered in detail in this paper.

4.3. *Other models of maintenance quality*

Alternative supplier models to model 1 might be used. We might allow the parameters of the component lifetime model to vary with the cost of a component. For example, we might allow the mixing parameter to vary. By adopting a functional form for the relationship between the mixing parameter and the cost of replacement, we could consider a continuum of quality levels—pay more to get higher quality components—and then determine the "optimum" quality level. A sensible parameterization of *p,* the mixing parameter, in these circumstances may be $p = p_0\{1+\delta(C-C_0)^{\kappa}\}^{-1}$, where C_0 is the minimum (baseline) expenditure on a component, and the decision is how much to spend C above the baseline ($C>C_0$). The mean delay time might be allowed to vary with cost of a component. Since a longer mean delay time implies that inspections will be more effective, and interpreting inspections as monitoring, allowing variation between maintenance suppliers in the delay time distribution parameters could be used to represent the cost of installing and using a condition monitoring device. For example, components from supplier A might follow a two state failure model (zero time in the defective state), while components from supplier B, with a monitoring device installed at a cost premium, would follow the three state failure model (with a non-zero delay time). The time to failure distributions for each type of component could be chosen to be the same. This would be modelled most simply by using, for supplier B, a time to defect arrival distribution F_X and a delay time distribution F_H, which would imply a time to failure distribution of $F_X * F_H(y) = \int_0^{\infty} F_H(y-x)\mathrm{d}F_X(x)$; $F_X * F_H$ could then also be used for the time to failure distribution of supplier A components, but without the possibility of a defective state. Comparison of the two suppliers can then be interpreted as assessing the economic and reliability benefits of condition monitoring in the manner of Al-Najjar (2007).

We do not explore these ideas further although it would be straightforward to do so within our framework. The modelling of false positives and negatives at inspection, on the other hand, is a more difficult mathematical problem, and we reserve this for future development.

5. Decision criteria

In all the models of quality we consider, the maintenance policy (paragraph 2, section 3) remains the same. A key assumption is that the system is renewed on replacement of the components. We can then use the renewal-reward theorem (Tijms, 1994) as justification for using the long-run cost per unit time as a decision criterion. This criterion is given by

$$C_{\infty}(T,\Delta,K) = \frac{\mathrm{E}(U)}{\mathrm{E}(V)}$$

where E(U) and E(V) are the expected cost per replacement cycle and the expected length of a replacement cycle, respectively. This cost can then be minimized with respect to K, Δ, and T.

For a maintained system, the times between operational failures are (approximately) exponentially distributed with mean $\mu = E(V)/\rho$ where ρ is the probability that a cycle (replacement interval) ends in failure (Scarf et al., 2005). The mean time between operational failures (MTBOF) provides a convenient reliability criterion. One might find that minimum cost policy that meets some reliability requirement expressed in terms of the MTBOF. A reliability constraint could also be expressed in terms the median time between operational failures, and an

expression for this is available (Scarf et al., 2005). The long-run cost and the MTBOF are used in the subsequent analysis.

6. Calculation of decision criteria with induced defects (model 2)

The calculation of the decision criteria is presented for model 2. Corresponding results for the two supplier model (model 1), for example, are obtained by setting q=0 in what follows, and repeating the calculations in each of the two supplier cases, with their differing values of mixing parameter and cost of preventive replacement.

6.1. *Notation*

K, Δ, T	number of inspections, inspection interval, age at preventive replacement (the decision variables).
q	the probability that a defect is induced at inspection.
X, f_X, F_X	component age at defect arrival and corresponding probability density and distribution function.
H, f_H, F_H	delay time from defect arrival to subsequent failure (time in defective state) and corresponding probability density and distribution function.
Y	age at failure, so that Y=X+H.
N	number of inspections in a renewal cycle until the first poor inspection (at which a defect is induced).
C_R	cost of preventive replacement
C_F	penalty cost of failure (>0), so that cost of failure replacement is C_R+C_F.
C_I	cost of inspection (<C_R)
U	cost of a renewal cycle (random variable)
V	length of a renewal cycle (random variable)
E(U)	expected cost of a renewal cycle
E(V)	expected length of a renewal cycle

6.2. *Expected cycle length calculation*

To calculate the long-run cost per unit time we proceed as follows. The cycle length is V and the cost over a cycle is U. Both U and V are random variables taking values that depend on how a replacement cycle ends. We consider four cases for how a replacement cycle ends.

Case1. Failure in an inspection interval (K>0)

First note that we do not consider the case K=0 here, because K is the number of inspections and if there are no inspections then a failure cannot arise in an inspection interval.

When K>0, in this case, the cycle length is the time to failure. Formally,

$$V = Y, \quad (i-1)\Delta < Y < i\Delta, \tag{1}$$

if $N > (i-1) \cap (i-1)\Delta < X \cap Y < i\Delta$ or if $N = (i-1) \cap (i-1)\Delta < X \cap H < \Delta$, ($i$=1,…,$K$). It then follows that the contribution to E(V) is

$$\sum_{i=1}^{K}(1-q)^{i-1}\int_{(i-1)\Delta}^{i\Delta}\int_{0}^{i\Delta-x}(x+h)\mathrm{d}F_H\mathrm{d}F_X+\sum_{i=2}^{K}(1-q)^{i-2}q\int_{(i-1)\Delta}^{\infty}\mathrm{d}F_X\int_{0}^{\Delta}[(i-1)\Delta+h]\mathrm{d}F_H \tag{2}$$

for K>1. The first term arises as a result of failures due to normal-use defects. The second term arises due to defects induced at inspection. In the case K=1 (one inspection only) the contribution to E(V) is simply

$$\int_0^{\Delta}\int_0^{\Delta-x}(x+h)\mathrm{d}F_H\mathrm{d}F_X \ .$$

This is the expected time to failure conditional on a normal-use defect arising in (0, Δ) and this defect causing a failure before the inspection, and is $\mathrm{prob}(X+Y<\Delta)$. There is no opportunity for a failure in the inspection interval due to an induced defect (as there is only one inspection) hence the second term in (2) disappears. It is informative to look at the cases K=2 and K=3 with regards to the second term in (2). When K=2, conditional on a defect due to normal-use not arising before the first inspection, there is a probability q that a defect is induced at the first inspection. This then gives rise to a failure at age Δ+h with probability $\mathrm{d}F_H$. This then leads to a probability of failure in the second inspection interval due to a defect induced at the first inspection given by

$$q\int_{\Delta}^{\infty}\mathrm{d}F_X\int_0^{\Delta}\mathrm{d}F_H \ , \tag{3}$$

and the contribution to E(V) in these circumstances is $q\int_{\Delta}^{\infty}\mathrm{d}F_X\int_0^{\Delta}(\Delta+h)\mathrm{d}F_H$. When K=3, the probability of failure in the second inspection interval due to a defect induced at the first inspection is as (3) above, and the probability of failure in the third inspection interval due to a defect introduced at the second inspection is $(1-q)q\int_{2\Delta}^{\infty}\mathrm{d}F_X\int_0^{\Delta}\mathrm{d}F_H$. Finally, we note that a failure due to an induced defect cannot arise in the ith inspection interval if a normal-use defect arises in the (i-1)th inspection interval. This is because the normal-use defect would either be detected at the (i-1)th inspection and the component replaced or have caused a failure before the (i-1)th inspection.

The expression (2) can also be derived by noting that the notional number of inspections until the first poor inspection (at which a defect is induced) has a geometric distribution, N~Ge(q), and then calculating the probability of the event (1). Implicit in this calculation and that above is the assumption that N, X, and Y statistically independent.

Case 2. Failure in the wear out phase $(K\Delta, T)$

Again the cycle length is the time to failure. Formally

$$V=Y, \quad K\Delta<Y<T,$$

if $N>K\cap K\Delta<X\cap Y<T$ or if $N=K\cap K\Delta<X\cap H<T-K\Delta$. Failure in the wear out phase may be due to a normal-use defect or an induced defect. Hence two terms arise in the contribution to E(V) which is given by

$$(1-q)^K\int_{K\Delta}^{T}\int_{0}^{T-x}(x+h)\mathrm{d}F_H\mathrm{d}F_X+(1-q)^{K-1}q\int_{K\Delta}^{\infty}\mathrm{d}F_X\int_{0}^{T-K\Delta}(K\Delta+h)\mathrm{d}F_H \tag{4}$$

(K>0). When K=0, this becomes

$$\int_0^T \int_0^{T-x} (x+h)\mathrm{d}F_H \mathrm{d}F_X \ .$$

Case 3. Replacement at inspection

The component will be replaced at the *i*th inspection if a normal-use defect arises in the *i*th inspection interval $((i-1)\Delta, i\Delta)$ or a defect is induced at the (*i*-1)th inspection, and this defect survives to the following inspection (at $i\Delta$). Formally:

$$V = i\Delta$$

if $N > (i-1) \cap (i-1)\Delta < X < i\Delta \cap \ i\Delta < Y$ or if $N = (i-1) \cap (i-1)\Delta < X \cap \ \Delta < H$ (*i*=1,…,*K*). Thus, for example, $V = \Delta$, and the component is replaced at first inspection, if $X < \Delta \cap \Delta < Y$. Further, $V = 2\Delta$ if $N > 1 \cap \Delta < X < 2\Delta \cap \ 2\Delta < Y$ or if $N = 1 \cap \ \Delta < X \cap \Delta < H$. If replacement is due to a normal-use defect then this occurs with probability

$$(1-q)^{i-1} \int_{(i-1)\Delta}^{i\Delta} [1 - F_H(i\Delta - x)]\mathrm{d}F_X \ .$$

If replacement is due to an induced defect then this occurs with probability

$$(1-q)^{i-2} q \int_{(i-1)\Delta}^{\infty} \mathrm{d}F_X [1 - F_H(\Delta)],$$

(*i*>1). Considering all inspections, the contribution to E(*V*) due to replacement at inspection is

$$\Delta \int_0^{\Delta} \overline{F}_H(\Delta - x)\mathrm{d}F_X + \sum_{i=2}^{K} i\Delta \left\{ (1-q)^{i-1} \int_{(i-1)\Delta}^{i\Delta} \overline{F}_H(i\Delta - x)\mathrm{d}F_X + (1-q)^{i-2} q \overline{F}_H(\Delta) \int_{(i-1)\Delta}^{\infty} \mathrm{d}F_X \right\},$$

K>1, where $\overline{F}_H(.) = 1 - F_H(.)$. When *K*=1, only the first term in above remains. When *K*=0 this case does not arise because there are no inspections.

Case 4. Replacement at age T

In the final case there are three possible events: a defect is induced at the final inspection and not before, and this survives until replacement; an induced defect does not arise (at any inspection) and a normal-use defect arises in $(K\Delta, T)$ but survives to *T*; an induced defect does not arise and no normal-use defect arises before *T*. Formally

$$V = T$$

if $N = K \cap K\Delta < X \cap T - K\Delta < H$ or if $N > K \cap K\Delta < X \cap T < Y$. The three events give rise to three terms in the expression for the contribution to E(*V*) given by

$$T \left\{ (1-q)^{K-1} q \overline{F}_H(T - K\Delta) \int_{K\Delta}^{\infty} \mathrm{d}F_X + (1-q)^K \int_{K\Delta}^{T} \overline{F}_H(T-x)\mathrm{d}F_X + (1-q)^K \int_T^{\infty} \mathrm{d}F_X \right\}, \quad (5)$$

(K>0). Note, in order for a defect to be induced at $K\Delta$ (final inspection), it is necessary that no defects are induced at earlier inspections and that a normal-use defect does not occur before $K\Delta$. When K=0, the expression (5) becomes

$$T\left\{\int_0^T \overline{F}_H(T-x)\mathrm{d}F_X + \int_T^\infty \mathrm{d}F_X\right\}.$$

The expected cycle length is then given by summing the contributions in each of the four cases:

$$\begin{aligned}
\mathrm{E}(V) &= \sum_{i=1}^{K}(1-q)^{i-1}\int_{(i-1)\Delta}^{i\Delta}\int_0^{i\Delta-x}(x+h)\mathrm{d}F_H\mathrm{d}F_X + \sum_{i=2}^{K}(1-q)^{i-2}q\int_{(i-1)\Delta}^{\infty}\mathrm{d}F_X\int_0^{\Delta}[(i-1)\Delta+h]\mathrm{d}F_H \\
&+ (1-q)^K\int_{K\Delta}^{T}\int_0^{T-x}(x+h)\mathrm{d}F_H\mathrm{d}F_X + (1-q)^{K-1}q\int_{K\Delta}^{\infty}\mathrm{d}F_X\int_0^{T-K\Delta}(K\Delta+h)\mathrm{d}F_H \\
&+ \Delta\int_0^{\Delta}\overline{F}_H(\Delta-x)\mathrm{d}F_X + \sum_{i=2}^{K}i\Delta\left\{(1-q)^{i-1}\int_{(i-1)\Delta}^{i\Delta}\overline{F}_H(i\Delta-x)\mathrm{d}F_X + (1-q)^{i-2}q\overline{F}_H(\Delta)\int_{(i-1)\Delta}^{\infty}\mathrm{d}F_X\right\} \\
&+ T\left\{(1-q)^{K-1}q\overline{F}_H(T-K\Delta)\int_{K\Delta}^{\infty}\mathrm{d}F_X + (1-q)^K\int_{K\Delta}^{T}\overline{F}_H(T-x)\mathrm{d}F_X + (1-q)^K\int_T^{\infty}\mathrm{d}F_X\right\}.
\end{aligned}$$

(K>1). When K=1, we have

$$\begin{aligned}
\mathrm{E}(V) &= \int_0^{\Delta}\int_0^{\Delta-x}(x+h)\mathrm{d}F_H\mathrm{d}F_X \\
&+ (1-q)\int_{\Delta}^{T}\int_0^{T-x}(x+h)\mathrm{d}F_H\mathrm{d}F_X + q\int_{\Delta}^{\infty}\mathrm{d}F_X\int_0^{T-\Delta}(\Delta+h)\mathrm{d}F_H \\
&+ \Delta\int_0^{\Delta}\overline{F}_H(\Delta-x)\mathrm{d}F_X \\
&+ T\left\{q\overline{F}_H(T-\Delta)\int_{\Delta}^{\infty}\mathrm{d}F_X + (1-q)\int_{\Delta}^{T}\overline{F}_H(T-x)\mathrm{d}F_X + (1-q)\int_T^{\infty}\mathrm{d}F_X\right\}.
\end{aligned}$$

When K=0, we have

$$\mathrm{E}(V) = \int_0^T\int_0^{T-x}(x+h)\mathrm{d}F_H\mathrm{d}F_X + T\left\{\int_0^T\overline{F}_H(T-x)\mathrm{d}F_X + \int_T^{\infty}\mathrm{d}F_X\right\}.$$

6.3 *Expected cost per cycle*

The cost incurred in a cycle is given by

$$U = \begin{cases}(i-1)C_{\mathrm{I}} + C_{\mathrm{R}} + C_{\mathrm{F}} & \text{if } (i-1)\Delta < V < i\Delta,\ i = 1,\ldots,K, \\ KC_{\mathrm{I}} + C_{\mathrm{R}} + C_{\mathrm{F}} & \text{if } K\Delta < V < T, \\ iC_{\mathrm{I}} + C_{\mathrm{R}} & \text{if } V = i\Delta,\ i = 1,\ldots,K, \\ KC_{\mathrm{I}} + C_{\mathrm{R}} & \text{if } V = T.\end{cases}$$

The four cases here correspond to the four cases described in the calculation of the expected cycle length: failure in the ith inspection interval (i=1,…K); failure in the wear-out phase; replacement at the ith inspection; and replacement at age T (at the end of the wear-out phase). The expected cost per cycle is then calculated in a similar manner to $\mathrm{E}(V)$, and it follows that

$$\mathrm{E}(U)=$$

$$\sum_{i=1}^{K}[(i-1)C_{\mathrm{I}}+C_{\mathrm{R}}+C_{\mathrm{F}}](1-q)^{i-1}\int_{(i-1)\Delta}^{i\Delta}\int_{0}^{i\Delta-x}\mathrm{d}F_H\mathrm{d}F_X+\sum_{i=2}^{K}[(i-1)C_{\mathrm{I}}+C_{\mathrm{R}}+C_{\mathrm{F}}](1-q)^{i-2}q\int_{(i-1)\Delta}^{\infty}\mathrm{d}F_X\int_{0}^{\Delta}\mathrm{d}F_H$$

$$+(KC_{\mathrm{I}}+C_{\mathrm{R}}+C_{\mathrm{F}})\left\{(1-q)^{K}\int_{K\Delta}^{T}\int_{0}^{T-x}\mathrm{d}F_H\mathrm{d}F_X+(1-q)^{K-1}q\int_{K\Delta}^{\infty}\mathrm{d}F_X\int_{0}^{T-K\Delta}\mathrm{d}F_H\right\}$$

$$+(C_{\mathrm{I}}+C_{\mathrm{R}})\int_{0}^{\Delta}\overline{F}_H(\Delta-x)\mathrm{d}F_X+\sum_{i=2}^{K}(iC_{\mathrm{I}}+C_{\mathrm{R}})\left\{(1-q)^{i-1}\int_{(i-1)\Delta}^{i\Delta}\overline{F}_H(i\Delta-x)\mathrm{d}F_X+(1-q)^{i-2}q\overline{F}_H(\Delta)\int_{(i-1)\Delta}^{\infty}\mathrm{d}F_X\right\}$$

$$+(KC_{\mathrm{I}}+C_{\mathrm{R}})\left\{(1-q)^{K-1}q\overline{F}_H(T-K\Delta)\int_{K\Delta}^{\infty}\mathrm{d}F_X+(1-q)^{K}\int_{K\Delta}^{T}\overline{F}_H(T-x)\mathrm{d}F_X+(1-q)^{K}\int_{T}^{\infty}\mathrm{d}F_X\right\}.$$

(K>1). When K=1, we have

$$\mathrm{E}(U)=(C_{\mathrm{R}}+C_{\mathrm{F}})\int_{0}^{\Delta}\int_{0}^{\Delta-x}\mathrm{d}F_H\mathrm{d}F_X$$

$$+(C_{\mathrm{I}}+C_{\mathrm{R}}+C_{\mathrm{F}})\left\{(1-q)\int_{\Delta}^{T}\int_{0}^{T-x}\mathrm{d}F_H\mathrm{d}F_X+q\int_{\Delta}^{\infty}\mathrm{d}F_X\int_{0}^{T-\Delta}\mathrm{d}F_H\right\}$$

$$+(C_{\mathrm{I}}+C_{\mathrm{R}})\int_{0}^{\Delta}\overline{F}_H(\Delta-x)\mathrm{d}F_X$$

$$+(C_{\mathrm{I}}+C_{\mathrm{R}})\left\{q\overline{F}_H(T-\Delta)\int_{\Delta}^{\infty}\mathrm{d}F_X+(1-q)\int_{\Delta}^{T}\overline{F}_H(T-x)\mathrm{d}F_X+(1-q)\int_{T}^{\infty}\mathrm{d}F_X\right\}.$$

When K=0, we have

$$\mathrm{E}(U)=(C_{\mathrm{R}}+C_{\mathrm{F}})\int_{0}^{T}\int_{0}^{T-x}\mathrm{d}F_H\mathrm{d}F_X+C_{\mathrm{R}}\left\{\int_{0}^{T}\overline{F}_H(T-x)\mathrm{d}F_X+\int_{T}^{\infty}\mathrm{d}F_X\right\}.$$

6.4. *Probability that a cycle ends in failure*

This probability is required in order to calculate (an approximation to) the mean time between operational failures. Considering cases 1 and 2 in the cycle length calculation, it follows from (2) and (4) that

$$\rho=\sum_{i=1}^{K}(1-q)^{i-1}\int_{(i-1)\Delta}^{i\Delta}\int_{0}^{i\Delta-x}\mathrm{d}F_H\mathrm{d}F_X+\sum_{i=2}^{K}(1-q)^{i-2}q\int_{(i-1)\Delta}^{\infty}\mathrm{d}F_X\int_{0}^{\Delta}\mathrm{d}F_H$$

$$+(1-q)^{K}\int_{K\Delta}^{T}\int_{0}^{T-x}\mathrm{d}F_H\mathrm{d}F_X+(1-q)^{K-1}q\int_{K\Delta}^{\infty}\mathrm{d}F_X\int_{0}^{T-K\Delta}\mathrm{d}F_H,$$

for K>1. For K=1, we have

$$\rho = \int_0^{\Delta} \int_0^{\Delta - x} \mathrm{d}F_H \mathrm{d}F_X + (1-q) \int_{\Delta}^{T} \int_0^{T-x} \mathrm{d}F_H \mathrm{d}F_X + q \int_{\Delta}^{\infty} \mathrm{d}F_X \int_0^{T-\Delta} \mathrm{d}F_H \, .$$

For K=0, we have

$$\rho = \int_0^{T} \int_0^{T-x} \mathrm{d}F_H \mathrm{d}F_X \, .$$

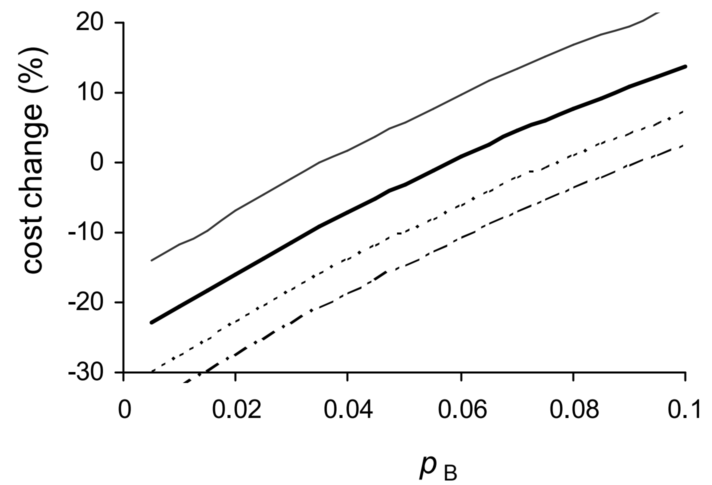
7. Pressure valve study

Application of the proposed maintenance quality models is now illustrated in the context of a pressure control valve in a gas transportation network. Such valves typically comprise the valve housing (socket), the internal parts of the valve (the component). We ignore an additional component, the valve pilot, that controls the valve through the upstream and downstream pressure difference and is typically external to the valve housing. The purpose of the valve is to maintain a constant required pressure for the consumer, and as pressure fluctuates upstream the valve will open and close in compensation. Failure of the valve leads to either a reduction in (or complete loss of) pressure to the consumer or a dangerous increase in pressure to the consumer. The valve is both inspected on a regular basis and the internal parts replaced regularly but less frequently. Early failures have been experienced, both following inspection and following replacement. The gas network operator can choose the supplier of maintenance and spare parts; typically this is a choice between the OEM (original equipment manufacturer) and a local competitor with lower maintenance costs to the operator. It is anticipated that the quality of the local supplier will be lower than that of the OEM, and that the quality of the supplier can manifest itself in a number of ways: through the quality of spare parts; the quality of the maintenance installation; the quality of the inspection process—the internal parts of the valve require careful set-up and such a set-up is required post inspection. The installation of monitoring equipment is an option for the operator.

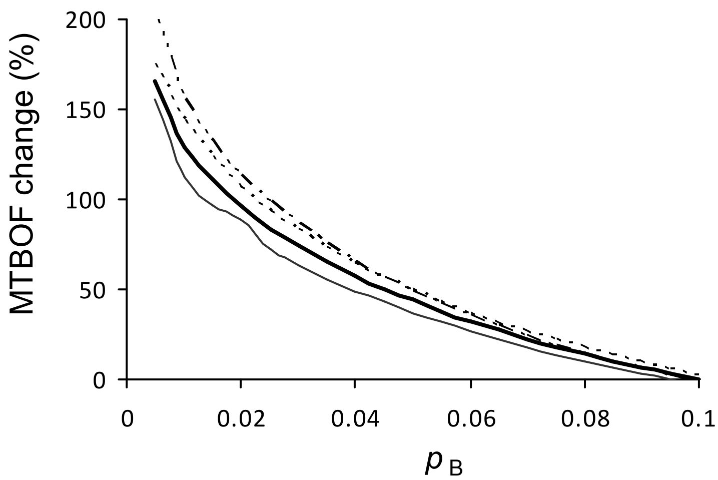
Failure data are not available for the estimation of the model parameters. Instead we use values consistent with engineering judgement, although the value of the mixing parameter is difficult to establish subjectively. Such a context is not untypical. In reality, data are often insufficient, inaccurate and incomplete, or where large records are available, systems have often evolved both during and beyond the time-frame of the data. Therefore, estimating model parameters using field-event data is often an unrealistic expectation. We argue that application of the models should focus on indicative results and what one can learn about a specific context in general terms. Our aim is more to provide a means for managers and engineers to gain insights about the systems they operate and less to do with the actual scheduling of maintenance activity for specific items. Consequently, the time to defect arrival of the values is assumed to be a mixture of Weibull distributions, with characteristic lives of 1 year and 6 years for the weak and the strong sub-populations respectively. The shape parameters are 2.5 and 5. This means that the sub-populations in the mixture are well separated. The mixing parameter is taken to be 0.10 (10% weak). The delay time distribution is exponential with mean 1/6 (years).

First we consider supplier quality model 1. Supplier A supplies components with proportion weak p_A=0.10 and replacement cost C_A=1 (C_A is the unit of cost). The effect of switching to supplier B (proportion weak components p_B, replacement cost C_A+C_B) is illustrated in figure 1.

Thus a 50% replacement cost premium would achieve overall cost savings if p_B~0.035 or less. Interestingly, if the decision maker is indifferent to the choice between A and B on the basis of cost (e.g. p_B =0.035, C_B =0.5 or p_B =0.06, C_B =0.3), the improvement (percentage increase) in the MTBOF is significant (60 and 40% respectively). As expected, the MTBOF is less sensitive to changes in the replacement premium cost, as the MTBOF depends on the policy costs only through the values of the decision variables. Table 1 considers sensitivity to values of the failure model and cost model parameters. As the penalty cost of failure increases, supplier B becomes more preferable. The decision appears to be relatively insensitive to the cost of inspection.



(a)



(b)

Figure 1. (a) Percentage change in long-run cost per unit time, (b) percentage change in MTBOF, if supplier B is adopted in place of supplier A as a function of p_B (proportion weak for supplier 2) with p_A=0.1 for various C_B (replacement cost premium for supplier B): —— C_B=0.5; ▬▬ C_B=0.3; ------ C_B=0.15; — — C_B=0.05. C_A=1.0 (unit of cost), C_F=9.0, λ=0.167 (2 months), c_I=0.05, β_1.=2.5, η_1=1.0, β_2=5.0, η_2=6.0. Unit of time is 1 year. In (b), decision variables are held at values that minimize the long-run cost.

Table 1. Optimum policy for various values of cost parameters and failure model with components supplied by supplier A (upper part), and components supplied by supplier B (lower part). p_A =0.1, C_A=1 (unit cost), λ=0.167 (2 months), β_1.=2.5, η_1=1.0, β_2=5.0, η_2=6.0 Unit of time is 1 year.

cost parameters				optimum values of decision variables			values of decision criteria	
c_I	c_F	p_B	C_B	T^*	Δ^*	K^*	C^*	$MTBOF$
Supplier A								
0.05	9.0			3.44	0.28	5	0.671	32.2
0.05	5.0			3.73	0.99	1	0.525	22.9
0.05	13.0			3.27	0.22	7	0.786	37.8
0.01	9.0			3.32	0.17	9	0.578	40.3
0.09	9.0			3.48	0.99	1	0.704	24.7
Supplier B								
0.05	9.0	0.01	0.10	3.16		0	0.470	76.5
0.05	5.0	0.01	0.10	3.52		0	0.409	54.7
0.05	13.0	0.01	0.10	2.96		0	0.518	91.4
0.01	9.0	0.01	0.10	3.16	1.01	1	0.469	79.9
0.09	9.0	0.01	0.10	3.16		0	0.470	76.5
0.05	9.0	0.05	0.10	3.33	0.99	1	0.588	39.5
0.05	5.0	0.05	0.10	3.65		0	0.476	31.2
0.05	13.0	0.05	0.10	3.16	0.43	3	0.682	47.8
0.01	9.0	0.05	0.10	3.97	0.12	32	0.484	81.6
0.09	9.0	0.05	0.10	3.34		0	0.594	35.9
0.05	9.0	0.01	0.50	3.34		0	0.595	64.5
0.05	5.0	0.01	0.50	3.86	1.78	2	0.532	45.5
0.05	13.0	0.01	0.50	3.13		0	0.65	78.8
0.01	9.0	0.01	0.50	3.34	1.01	1	0.593	66.8
0.09	9.0	0.01	0.50	3.34		0	0.595	64.5
0.05	9.0	0.05	0.50	3.49	0.99	1	0.710	36.6
0.05	5.0	0.05	0.50	3.84		0	0.588	28.2
0.05	13.0	0.05	0.50	3.29	0.43	3	0.811	44.8
0.01	9.0	0.05	0.50	4.28	0.11	39	0.587	74.4
0.09	9.0	0.05	0.50	3.49		0	0.717	33.6

Now consider supplier quality model 2: induced defects. First we consider the effect of induced defects in isolation—one supplier model—and ask: when does it become uneconomic to inspect and what then is the effect on reliability? In table 2, the value of q is determined such that the long-run cost of a zero-inspection policy (K=0) is the same as that for a policy with a single inspection (K=1). For this value of q, policies with more inspections will necessarily cost more in the long-run. This is because of the additional opportunities for defect induction. We determine the critical value of q when the cost of inspection is very small (5×10^{-6}) and for the base case value (5×10^{-2}). Notice that the size of the change in the critical value of q over these two cases does not depend on p. This is as expected since if one inspection is carried out, then the cost of

inspection changes from 5×10^{-6} to 5×10^{-2}, that is ~0.05. This is balanced by a change in the cost due to the failure that will (almost always) be induced with probability *q*. With the cost consequences of failure equal to $C_R+C_F=10$, then the corresponding change in *q* will be ~0.005 as observed in table 2.

The effect of *q* and hence defect induction at inspection is large (comparing line 1 in table 1 with lines 4, 5 and 6 in table 2). Small values of *q* make inspection uneconomic and increase overall costs and decrease reliability significantly.

It appears that the effect of defect induction is one order of magnitude greater than the effect of the mixing proportion. On first sight, this dramatic effect warranted further investigation, and the effect of inspection on cost and reliability for a moderate value of *q* (*q*=0.05) is investigated in table 3. For all the cases therein the effect of inspection (i.e. *q*) on the time to replacement is small, T^*~3-4. The effect of inspection on cost and reliability is however large, as stated. In particular, consider the case *p*=0. Broadly speaking, when *K*=0, MTBOF=120 (approx), that is 1 failure for every 40 replacements (approx), as a replacement interval is approximately 3 years in length. Now when *K*=2 (say), there is approximately a 10% chance of introducing a defect in any replacement interval (*q*=0.05). If the delay time is small, such a defect will result in failure nearly always, and so 10% (approx) of replacement intervals will have a failure: 1 in every 10, which corresponds to an MTBOF of 30 years approximately. So with *q*=0.05, we see an approximate 4 fold change in MTBOF when moving from no inspections (*K*=0) to two inspections (*K*=2). The effect of *q* is dramatic because, without inspection, failures are rare (Pr(failure)~1/40=0.025<*q*). Thus any maintenance intervention that induces a problem, with a probability similar to the inherent failure probability (without maintenance), is guaranteed to be sub-optimal.

Table 2. Minimum cost replacement policy when *K*=0 and *K*=1 (zero or one inspection) for single supplier model for various probabilities of defect induction at inspection, *q*, and mixing parameter *p*. C_R=1 (unit cost), λ=0.167 (2 months), β_1.=2.5, η_1=1.0, β_2=5.0, η_2=6.0, $c_I=5\times10^{-6}$, $c_F=9$. Unit of time is 1 year.

			values of decision variables			values of decision criteria	
q	p	c_I	T^*	Δ^*	K	C^*	$MTBOF$
0.0075	0.05	5×10^{-6}	3.296	0.990	1	0.563	36.70
0.0023	0.05	5×10^{-2}	3.296	0.989	1	0.563	38.98
	0.05		3.296		0	0.563	36.44
0.0153	0.1	5×10^{-6}	3.497	0.990	1	0.721	22.09
0.0101	0.1	5×10^{-2}	3.497	0.990	1	0.721	22.88
	0.1		3.497		0	0.721	21.92
0.0400	0.25	5×10^{-6}	3.975	0.997	1	1.227	9.95
0.0349	0.25	5×10^{-2}	3.975	1.001	1	1.227	10.09
	0.25		3.975		0	1.227	9.85
0.0897	0.5	5×10^{-6}	4.642	1.014	1	2.327	4.63
0.0847	0.5	5×10^{-2}	4.642	1.019	1	2.327	4.67
	0.5		4.642		0	2.327	4.58

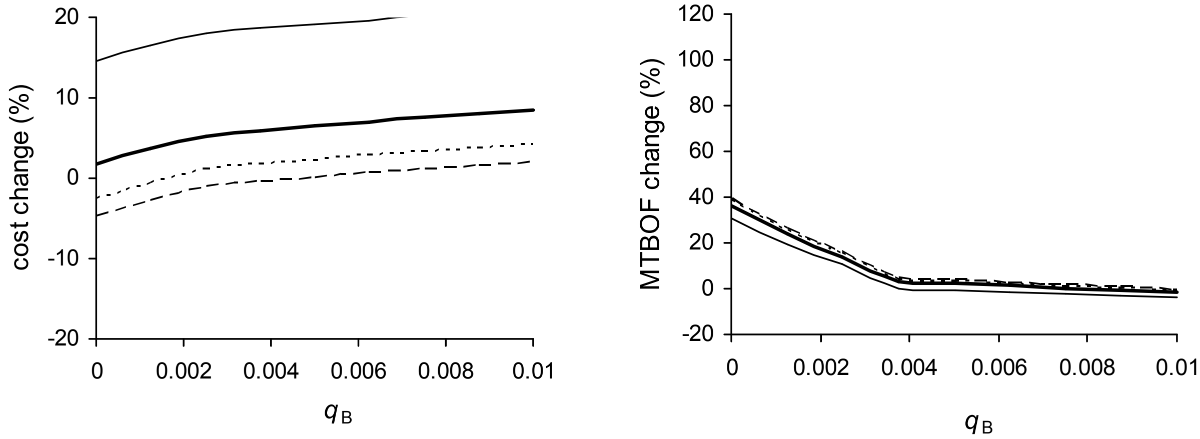
Table 3. Minimum cost policies as a function of the number of inspections K for q=0.05, for various values of the mixing parameter p. One supplier model, C_A=1.0 (unit of cost), λ=0.167 (2 months), β_1.=2.5, η_1=1.0, β_2=5.0, η_2=6.0, $c_F = 9$. Unit of time is 1 year.

		values of decision variables			values of decision criteria	
p	*q*	*T**	*Δ**	*K*	*C**	*MTBOF*
0.15	0.05	3.671		0	0.882	15.68
0.15	0.05	3.759	1.023	1	0.973	13.93
0.15	0.05	3.866	0.625	2	1.094	12.02
0.15	0.05	3.958	0.416	3	1.205	10.70
0.15	0.05	4.033	0.283	4	1.303	9.80
0.1	0.05	3.497		0	0.721	21.92
0.1	0.05	3.624	1.039	1	0.836	17.61
0.1	0.05	3.636	1.818	2	0.969	16.71
0.1	0.05	3.790	1.263	3	0.978	14.09
0.1	0.05	3.981	0.995	4	1.090	12.05
0.05	0.05	3.296		0	0.563	36.44
0.05	0.05	3.478	1.090	1	0.704	23.74
0.05	0.05	3.499	1.750	2	0.724	23.04
0.05	0.05	3.696	1.232	3	0.850	17.52
0.05	0.05	3.889	0.972	4	0.973	14.21
0	0.05	3.049		0	0.406	116.83
0	0.05	3.081	3.081	1	0.422	111.80
0	0.05	3.362	1.681	2	0.584	36.20
0	0.05	3.594	1.198	3	0.729	22.82
0	0.05	3.794	0.948	4	0.862	17.13

We now consider a two supplier model with induced defects in which, for supplier A, q_A>0 and C_R=C_A=1, and for supplier B, q_B<q_A, and C_R=C_A+C_B. The value of q_A is specified such that with supplier A the operator will be indifferent to carrying out inspection. Such a value depends on the value of p_A and is indicated in table 2. The interpretation of this scenario is as follows: both suppliers use the same components, but supplier A is cheaper but their maintenance is of poorer quality. So typically supplier B might be the OEM, supplier A might be a local maintainer who still uses OEM spare parts. Hence, figures 2 and 3 show the effect of q_B and C_B on the overall cost and reliability when switching supplier. When the mixing parameter takes a moderate value, p_A= p_B=0.1, figure 2, the gains from switching maintenance supplier can only be justified when the additional costs associated with supplier B are not too large. Here q_A =0.01 so the induction probability for supplier A is reasonably small. Such gains from switching supplier will only be greater than those shown in figure 2 if q_A >0.01 and if A is inspecting sub-optimally—that is, if they are inspecting (K>0). This is because, in these circumstances (p_A=0.1, q_A >0.01), a zero-inspection policy is optimal (table 2). Note that the reliability (MTBOF) is insensitive to changes in supplier B costs as we would expect. The slight variation in the reliability here is due to small changes in the values of decision variables with changing C_B. The gains are greater when the

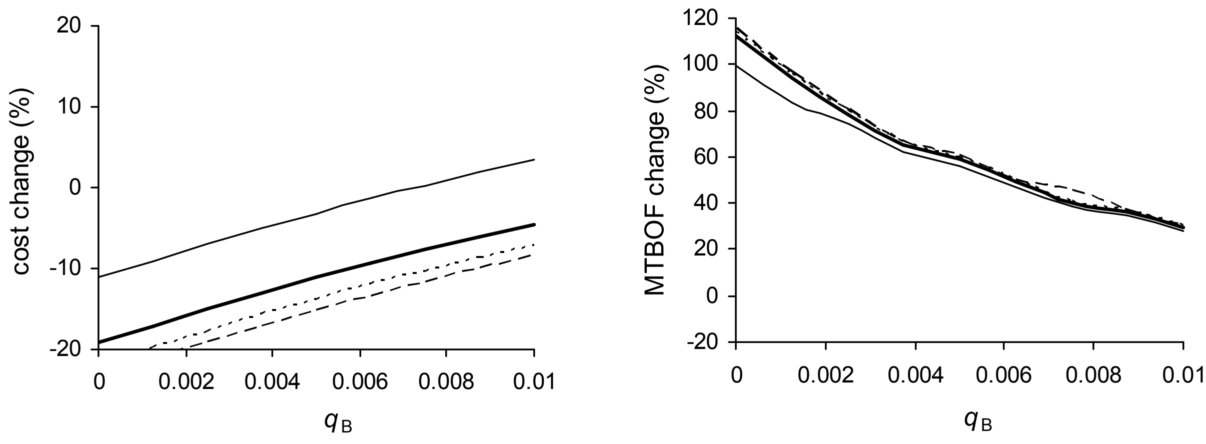
mixing parameter takes a larger value, $p_A = p_B = 0.25$, figure 3. The corresponding critical value of q_A, such that with supplier A the operator will be indifferent to carrying out inspection, is then 0.035, and this is the value used in the calculations in figure 3.

On the whole figures 2 and 3 again emphasize that poor inspection is a much bigger problem than poor component quality, where poor inspection here implies defect induction.



(a) (b)

Figure 2. (a) Percentage change in long-run cost per unit time, and (b) percentage change in MTBOF, if supplier B is adopted in place of supplier A as a function of q_B (probability of inducing a defect for supplier B) with $p_A = p_B = 0.1$ and $q_A = 0.01$ (the critical value from table 2). For various C_B (replacement cost premium with supplier B): —— C_B=0.5; ▬▬ C_B=0.2; ------ C_B=0.1; — — C_B=0.05. C_A=1.0 (unit of cost), C_F=9.0, λ=0.167 (2 months), β_1.=2.5, η_1=1.0, β_2=5.0, η_2=6.0. c_I=0.05. Unit of time is 1 year. In (b) and (d), decision variables are held at values that minimize the long-run cost.



(c) (d)

Figure 3. As figure 2 but with $p_A = p_B = 0.25$ and $q_A = 0.035$ (the critical value from table 2).

We do not consider false positives and negatives at inspection in this paper. These are further issues in inspection quality, although our instinct is that their effects will not be as large as poor inspection through defect induction. In fact, false positives, that report a defect when the

component is good, would tend to lead to increased costs due to the execution of additional, unnecessary replacements. Decreasing reliability would also tend to occur if the mixing parameter is relatively large. This is because a good component may be unnecessarily replaced with a poor component. False negatives, that report a component as good when it is in fact defective, may not be such a big problem provided inspection costs are not too large—this is because one can merely inspect more often to mitigate against false negatives. Of course, the possibility of defect induction would complicate matters here significantly.

8. Discussion

In this paper we propose some simple models of supplier choice in preventive maintenance, including inspection and replacement. Competing suppliers may supply replacement components of differing quality and cost. They may carry out maintenance interventions with differing quality and cost. We model component lifetimes as a mixture, with two sub-populations in the mixture, one corresponding to short lifetimes (weak) and the other to long lifetimes (strong). The quality of component supply is modelled through the probability that the component supplied is weak (the mixing parameter). In our model, a poor quality component is analogous to incorrect installation, and so the quality of replacement can be considered. The quality of inspection maintenance is modelled by allowing for the possibility of defect induction at inspection. The mixing parameter and the defect induction parameter are then allowed to vary between maintenance suppliers in a way that reflects the costs associated with those suppliers. Thus, our model mirrors the context in which an operator can choose between maintenance contractors. These contractors may include the original equipment manufacturer and competing local suppliers. Large variations in cost and quality can be particularly apparent for complex equipment located in developing countries.

Typical models of quality in maintenance to date have been concerned with the effect of maintenance on the quality of production output. Our notion of quality in maintenance here is different. We are concerned with the quality of maintenance per se. To our knowledge, this notion of quality in maintenance has not been the subject of modelling until now. Our study has developed from earlier work on optimizing preventive maintenance for systems when component lifetimes arise from a mixture. We proposed a hybrid policy with an early inspection phase to mitigate against the likelihood of installation of a weak component (Scarf et al, 2009). Such an inspection phase acts like an operational burn-in period. Of course, rather than perform early inspections, one might attempt to improve the quality of components. This idea motivates our work here.

The effect of the defect induction parameter on the overall costs and reliability appears to be significantly greater than the effect of the mixing parameter. Inspections aim to improve reliability and reduce costs by detecting defects in weak components early in the replacement cycle. However, if the probability of inducing a defect at inspection is not small, we show that the inspections themselves can have a greater impact on system reliability and overall cost than the quality of the components, even when the quality of the components is relatively poor (large value of the mixing parameter). System operators who introduce inspection regimes to mitigate against early failures may be doing more harm than good, even when there is only the slightest chance that an inspection induces a defect. Furthermore, the intuitive response to an increasing frequency of

failure, in which a maintainer increases the frequency of inspection, may further exacerbate matters—the more one tries to reduce faults, the worse the final behavior of the system.

Thus, indirectly, our work attempts to model human failings in the maintenance process. While the majority of maintenance studies are concerned with the failure behaviour of equipment which is a result of the effect of exposure time on the equipment, these equipments form part of production and other systems which are typically subject to interference of man, not only to operate these systems, but also to re-establish the operational condition through maintenance action. The effect of this interference may be much greater than is typically assumed.

There are limitations in the work presented here. The idea of a system comprising a single component is a rather simplistic view. Model parameters will in general be very difficult to estimate. The maintenance policies we consider are also static in nature. When optimizing the maintenance of a particular, complex system in real time, one would ideally use a dynamic policy that updates the decision as information from inspections arises, but this is often an unrealistic expectation. On the other hand, one can gain insights from the quantitative study of artificial systems or historical systems, and then use these insights to guide decision-making about contemporary systems. Thus, our concern in this paper is with the indicative effects of variation in quality of supplier on inspection and replacement policy.

Other studies both within our framework and beyond could be considered. For example, the cost-benefit of introducing condition monitoring may be evaluated within our framework. The effect of false positives (replacement at inspection of a component when it is not defective) and false negatives (non-replacement at inspection of a component when it is defective) requires further modelling development. Induced defects might also be considered as an additional failure mode in a competing risks model.

Acknowledgement

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