A smashing analysis of badminton scoring systems

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A Smashing Analysis of Badminton Scoring Systems

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Summary

The International Badminton Federation (IBF) recently introduced some rule changes to make the game faster and more entertaining. These affect how players score points and win games: the number of points per game is increased for all matches; players can score on their opponents' serves; doubles pairs only have one serve rather than two; setting is replaced by a bounded two-point advantage.

We assess the overall fairness and discriminatory ability of both systems by applying combinatorial, probabilistic and simulation methods to extrapolate known probabilities of winning individual rallies into probabilities of winning games and then matches. We also measure how well the rule changes meet the IBF's aspirations by comparing the numbers of rallies per game and the scoring patterns within each game under both sets of rules.

We then relax the above model assumptions by considering frequentist methods for estimating the probability that a singles player or doubles pair will win an individual rally, as this parameter is always unknown in practice. For demonstrating our results, we use actual performance data from the 2006 Commonwealth Games to assess and compare the two scoring systems – did all the right players win?

Finally, we resolve the difficulties of parameter estimation by developing subjective Bayesian methods for specifying the probabilities of winning individual rallies. We then describe how to propagate this information with observed data to determine posterior predictive distributions for match outcomes, so enabling us to predict match outcomes for given scenarios before and during play.

Keywords:
badminton, scoring systems, simulation, subjective probability, predictive distribution.

1. Introduction

Forgive the pun in the title – I attribute this to my niece, Rebecca, who kindly assisted with data collection and algebraic calculations for this research. According to the Wikipedia website (2006), the sport of badminton originated about two thousand years ago in ancient Greece and spread to China, India, Japan and Thailand. In 1893, the Badminton Association of England published the first proper set of rules and, in 1934, the International Badminton Federation (IBF) was established. It is now established as the game's governing body, involving 152 countries and hosting over 100 annual tournaments.

Whilst the academic literature has addressed some aspects of badminton, it has paid little attention to the scoring systems used. Instead, most of the published articles consider the techniques, physiology and psychology of the sport, including papers by Yuan *et al*. (1995), Sakurai and Ohtsuki (2000), Blomqvist *et al*. (2001), Lee and Gu (2004) and Tsai *et al*. (2006). However, the paper by Seve and Poizat (2005) does address scoring systems in another racket sport, namely table tennis.

There are five recognised badminton events: women's singles, men's singles, mixed doubles, women's doubles and men's doubles. Until 2002, the rules had remained almost unchanged. Then the IBF introduced a series of experimental rule changes in order to make the game faster and more exciting. In 2006, some new rules were introduced for the Commonwealth Games in Melbourne, Australia, and this article evaluates the pros and cons of these variations. Table 1 summarizes the main differences between the old, experimental and new rules.

	old rules to 2002	experimental rules 2002		from 2002 to 2006	new rules from 2006
games per match	3	5	3	3	3
points per game	15 or 11	7	11 or 15	15 or 11	21
close endgames	setting options	setting option	setting option	setting option	two-point lead
score points on	own serve	own serve	own serve	own serve	any serve
serves in doubles	2	2	2	2	1

Table 1: comparative summary of old, experimental and new badminton rules.

Under the old rules (before 2002), the first team (player or pair) to win fifteen points (for all events except women's singles) or eleven points (for women's singles only) wins a game. Under the new rules (after 2006), the first team to win twenty-one points (for all events) wins a game. In both cases, the first team to win two games wins the match. However, there are other differences between these rules.

Under the old rules, teams only score points when they win rallies on their own serves and doubles pairs have two serves per cycle. Under the new rules, teams score points when they win rallies on either team's serves and doubles pairs only have one serve per cycle. Moreover, close games are resolved differently. The old rules involve setting, which we describe in Section 2, whereas the new rules require the winner to establish a two-point advantage subject to a maximum of thirty points.

Inspired by some similar, though easier, analyses for the sport of tennis described by Stewart (1991), this paper starts by assuming that a given team has a constant probability of winning any particular rally in any particular match. We investigate the criteria under which teams should opt to set games in different scenarios under the old rules. Based on combinatorics, probability theory and the solution of polynomial equations by analytical and

numerical methods, we evaluate prescriptive thresholds for all events. We then combine these into simulation programs to determine the probability that a given team wins any particular game under the old rules.

We derive algebraic expressions for this probability under the new rules and compare these results with the simulation output to assess whether the two sets of rules are comparable in deciding the winners of matches. We also consider to what extent the new rules meet the IBF's intentions of making badminton matches more exciting. The main criterion for measuring this performance is the number of rallies per game, whose mean and standard deviation we evaluate by computer simulation.

As the probabilities of winning rallies are unknown in practice, we investigate frequentist methods for estimating these parameters. We then analyse the finals of the badminton competition at the 2006 Commonwealth Games in Melbourne (Australia) for empirical confirmation and to determine whether the same players would have won under both sets of rules. Our results show that the new rules do achieve the desired aims, at the expense of losing the uniqueness and tradition of the old rules.

Finally, we resolve the difficulties with estimation by investigating subjective Bayesian methods for specifying the probability that a singles player or doubles pair will win an individual rally, based upon suitable procedures for prior elicitation. We describe how to propagate this information with observed data to determine posterior predictive distributions for match outcomes and discuss how to make use of these for forecasting results and making decisions before and during play.

2. Setting Decisions

Our analysis begins by investigating a set of isolated decision problems that arise in setting, which is the procedure for settling close endgames under the old rules (before 2002).

First consider women's singles, where the first player to score eleven points wins a game. If the score reaches nine all (9-9), the player who reached nine first has the option of "playing through" or "setting to three". If she chooses to play through, then the game continues uninterrupted until one player reaches eleven points. If she chooses to set to three, the scores become love all (0-0) and play resumes until one player reaches three points.

If the score reaches ten all (10-10), the player who reached ten first has the option of "playing through" or "setting to two". If she chooses to play through, then the game continues uninterrupted until one player reaches eleven points. If she chooses to set to two, the scores become love all (0-0) and play resumes until one player reaches two points. Notice that the option of setting is available at 10-10 even if a previous option of setting was declined at 9-9.

Similar options of setting at are available for the other four events, where the first team to score fifteen poins wins a game. At 13-13, the team that scores thirteen points first may opt to play through or set to five. At 14-14, the team that scores fourteen points may opt to play through or set to three. Again, the option of setting is available at 14-14 even if a previous option of setting was declined at 13-13.

A natural question to ask is, "if a close endgame arises during play, should a team opt to play through or set?" Good players usually opt to set, but is this the right decision? We now demonstrate how to answer these questions and stumble across some interesting results. In order to proceed, we assume that team A has a constant probability θ of winning any rally against team B and that the outcomes of all rallies are independent.

2.1. Women's Singles Events

Define $P(A|B)$ to be the probability that player A wins the next point if player B is serving. There is an infinite number of ways for this event to occur because she can only score a point on her own serve. These arise as follows:

A wins a rally to gain serve and
A wins a rally to win the point;

A wins a rally to gain serve,
B wins a rally to gain serve,
A wins a rally to gain serve and
A wins a rally to win the point;

A wins a rally to gain serve,
B wins a rally to gain serve,
A wins a rally to gain serve,
B wins a rally to gain serve,
A wins a rally to gain serve and
A wins a rally to win the point;

and so on. These events are mutually exclusive, so we can sum their probabilities to obtain the geometric progression

$$\begin{aligned} P(A|B) &= \theta^2 + \theta(1-\theta)\theta^2 + \theta(1-\theta)\theta(1-\theta)\theta^2 + \ldots \\ &= \theta^2 \sum_{i=0}^{\infty} \{\theta(1-\theta)\}^i = \theta^2 \frac{1}{1-\theta(1-\theta)} = \frac{\theta^2}{\theta^2 - \theta + 1}. \end{aligned} \tag{1}$$

Similarly, player A can win the next point if she is serving, according to any term in this infinite sequence of rally winners: A; BAA; $BABAA$; *etc*. The corresponding probability of this event is given by

$$\begin{aligned} P(A|A) &= \theta + (1-\theta)\theta^2 + (1-\theta)\theta(1-\theta)\theta^2 + \ldots \\ &= \theta \sum_{i=0}^{\infty} \{\theta(1-\theta)\}^i = \theta \frac{1}{1-\theta(1-\theta)} = \frac{\theta}{\theta^2 - \theta + 1}. \end{aligned} \tag{2}$$

By symmetry, the other two probabilities of similar events are

$$P(B|A) = \frac{(1-\theta)^2}{\theta^2 - \theta + 1} \tag{3}$$

and

$$P(B|B) = \frac{1-\theta}{\theta^2 - \theta + 1}\,. \tag{4}$$

We are now able to evaluate the probabilities that a player wins any particular endgame scenario, if she opts to play through and if she opts to set.

2.1.1. Women's Singles 10-10

Firstly, suppose that player B is serving and levels the score at 10-10, so player A can choose to play through or set to two points. Figure 1 illustrates that there is only one pattern of scoring

(10-10) $\rightarrow$ (11-10)

if she opts to play through, but that there are three mutually exclusive patterns of scoring

(0-0) $\rightarrow$ (1-0) $\rightarrow$ (2-0)
(0-0) $\rightarrow$ (1-0) $\rightarrow$ (1-1) $\rightarrow$ (2-1)
(0-0) $\rightarrow$ (0-1) $\rightarrow$ (1-1) $\rightarrow$ (2-1)

if she opts to set to two.

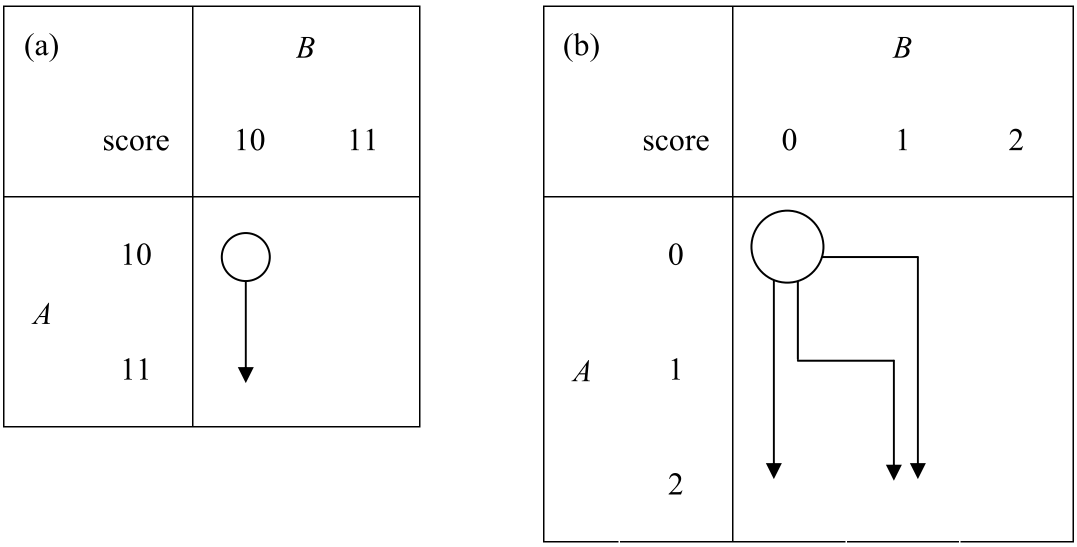

Figure 1: scoring patterns for player A to win endgame at 10-10 in women's singles when (a) playing through and (b) setting to two.

Defining $\phi_1(\theta)$ and $\phi_2(\theta)$ to be the probabilities that player A wins the endgame if she plays through and sets to two respectively, it is clear from above and from Equation (1) that

$$\phi_1(\theta) = P(A|B) = \frac{\theta^2}{\theta^2 - \theta + 1}\,. \tag{5}$$

We also have

$$\phi_2(\theta) = P(A|B)P(A|A) + P(A|B)P(B|A)P(A|B) + P(B|B)P(A|B)P(A|A)$$
$$= \frac{\theta^3}{(\theta^2-\theta+1)^2} + \frac{\theta^4(1-\theta)^2}{(\theta^2-\theta+1)^3} + \frac{\theta^3(1-\theta)}{(\theta^2-\theta+1)^3} = \frac{\theta^3(\theta^3-\theta^2-\theta+2)}{(\theta^2-\theta+1)^3} \tag{6}$$

from above and from Equations (1) to (4).

To determine what values of θ lead to recommendations of playing through and setting to two points at 10-10 in women's singles games, we investigate the conditions under which $\phi_1(\theta) = \phi_2(\theta)$. Consider the difference

$$\phi_1(\theta) - \phi_2(\theta) = \frac{\theta^2}{\theta^2-\theta+1} - \frac{\theta^3(\theta^3-\theta^2-\theta+2)}{(\theta^2-\theta+1)^3} = \frac{\theta^2(1-\theta)(\theta^2-3\theta+1)}{(\theta^2-\theta+1)^3}. \tag{7}$$

Clearly, there are trivial solutions at $\theta = 0$, where $\phi_1(0) = \phi_2(0) = 0$, and at $\theta = 1$, where $\phi_1(1) = \phi_2(1) = 1$. To look for solutions in the interval $0 < \theta < 1$, Equation (7) requires us to solve the quadratic equation

$$\theta^2 - 3\theta + 1 = 0. \tag{8}$$

The two roots are given by

$$\theta = \frac{3 \pm \sqrt{5}}{2} \approx \begin{cases} 0 \cdot 382 \\ 2 \cdot 62 \end{cases} \tag{9}$$

but only the first of these, corresponding to the negative root, is in the feasible range. A simple numerical check leads to the conclusion that player A should play through if $\theta < 0 \cdot 382$ and should set to two points if $\theta > 0 \cdot 382$, as illustrated in Figure 2.

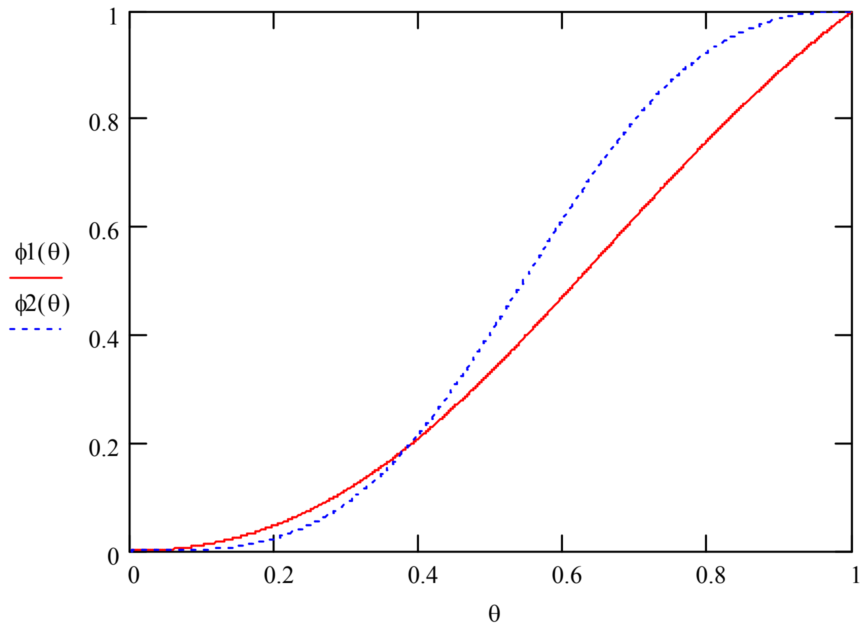

Figure 2: probabilities of winning when playing through, $\phi_1(\theta)$, and setting to two points, $\phi_2(\theta)$, at 10-10 in women's singles games for different values of θ.

We now determine the values of θ, the assumed constant probability that player A wins any particular rally against player B, which correspond to local maxima for the absolute difference in probabilities of winning the endgame when playing through and setting to two, $|\phi_1(\theta)-\phi_2(\theta)|$. From Equation (7) and after some algebraic simplification,

$$\frac{d}{d\theta}\{\phi_1(\theta)-\phi_2(\theta)\}=\frac{\theta(\theta^5-6\theta^4+3\theta^3+12\theta^2-11\theta+2)}{(\theta^2-\theta+1)^4} \qquad (10)$$

with non-trivial, feasible roots at $\theta \approx 0\cdot 26$ and $\theta \approx 0\cdot 72$, as calculated numerically using Mathcad software. These values agree with Figure 2 and imply that the importance of opting to play through is greatest at $\theta \approx 0\cdot 26$, where $|\phi_1(\theta)-\phi_2(\theta)| \approx 0\cdot 03$ from Equation (7), and the importance of opting to set to two is greatest at $\theta \approx 0\cdot 72$, where $|\phi_1(\theta)-\phi_2(\theta)| \approx 0\cdot 18$.

2.1.2. Women's Singles 9-9

We now turn our attention to the other setting opportunity in women's singles games. This arises when the score reaches 9-9 with player B serving, in which case player A can opt to play through to eleven or set to three points. If she chooses to play through, the scoring patterns that lead to her winning the game are

(9-9) $\rightarrow$ (10-9) $\rightarrow$ (11-9)
(9-9) $\rightarrow$ (10-9) $\rightarrow$ (10-10) $\rightarrow$ A can set
(9-9) $\rightarrow$ (9-10) $\rightarrow$ (10-10) $\rightarrow$ B can set

and we assume that players choose their optimal setting strategies at 10-10 according to our earlier analysis. Defining $\phi_3(\theta)$ to be the probability that player A wins the endgame from 9-9 if she opts to play through to eleven points, we note that $\phi_3(\theta)$ is a piecewise function whose intervals reflect the discrete changes in setting decisions at 10-10. We need to consider three intervals separately in this case, as follows.

Firstly in the interval $0 < \theta < 0\cdot 382$, player A will play through to eleven points at 10-10 if given the opportunity because $\theta < 0\cdot 382$, whereas player B will set to two points because $1-\theta > 0\cdot 382$. Consequently, the probability that player A wins the endgame if she opts to play through at 9-9 is

$$\begin{aligned}\phi_{3(1)}(\theta) &= P(A|B)P(A|A) + P(A|B)P(B|A)\phi_1(\theta) + P(B|B)P(A|B)\{1-\phi_2(1-\theta)\} \\ &= \frac{\theta^3}{(\theta^2-\theta+1)^2} + \frac{\theta^4(1-\theta)^2}{(\theta^2-\theta+1)^3} + \frac{\theta^4(1-\theta)(2\theta^3-3\theta^2+\theta+1)}{(\theta^2-\theta+1)^5} \\ &= \frac{\theta^3(\theta^7-3\theta^6+3\theta^5+\theta^4-3\theta^3+2\theta^2-\theta+1)}{(\theta^2-\theta+1)^5}\end{aligned} \tag{11}$$

from Equations (1) to (6). Secondly in the interval $0\cdot 382 < \theta < 0\cdot 618$, both players will set to two points at 10-10 if given the opportunity because $\theta > 0\cdot 382$ and $1-\theta > 0\cdot 382$. Consequently, the probability that player A wins the endgame if she opts to play through at 9-9 is

$$\begin{aligned}\phi_{3(2)}(\theta) &= P(A|B)P(A|A) + P(A|B)P(B|A)\phi_2(\theta) + P(B|B)P(A|B)\{1-\phi_2(1-\theta)\} \\ &= \frac{\theta^3}{(\theta^2-\theta+1)^2} + \frac{\theta^5(1-\theta)^2(\theta^3-\theta^2-\theta+2)}{(\theta^2-\theta+1)^5} + \frac{\theta^4(1-\theta)(2\theta^3-3\theta^2+\theta+1)}{(\theta^2-\theta+1)^5} \\ &= \frac{\theta^3(\theta^7-2\theta^6-3\theta^5+14\theta^4-16\theta^3+8\theta^2-2\theta+1)}{(\theta^2-\theta+1)^5}\end{aligned} \tag{12}$$

from Equations (1) to (6). Thirdly in the interval $0\cdot 618 < \theta < 1$, player A will set to two points at 10-10 if given the opportunity because $\theta > 0\cdot 382$, whereas player B will play through to eleven points because $1-\theta < 0\cdot 382$. Consequently, the probability that player A wins the endgame if she opts to play through at 9-9 is

$$\begin{aligned}\phi_{3(3)}(\theta) &= P(A|B)P(A|A) + P(A|B)P(B|A)\phi_2(\theta) + P(B|B)P(A|B)\{1-\phi_1(1-\theta)\} \\ &= \frac{\theta^3}{(\theta^2-\theta+1)^2} + \frac{\theta^5(1-\theta)^2(\theta^3-\theta^2-\theta+2)}{(\theta^2-\theta+1)^5} + \frac{\theta^3(1-\theta)}{(\theta^2-\theta+1)^3} \\ &= \frac{\theta^3(\theta^7-2\theta^6-2\theta^5+12\theta^4-17\theta^3+13\theta^2-6\theta+2)}{(\theta^2-\theta+1)^5}\end{aligned} \tag{13}$$

from Equations (1) to (6). Collating these results gives the probability that player A wins the endgame from 9-9 if she opts to play through to eleven points as

$$\phi_3(\theta) = \begin{cases} \phi_{3(1)}(\theta); & 0 < \theta < 0\cdot 618 \\ \phi_{3(2)}(\theta); & 0\cdot 382 < \theta < 0\cdot 618 \\ \phi_{3(3)}(\theta); & 0\cdot 618 < \theta < 1 \end{cases} \tag{14}$$

in terms of θ, the assumed constant probability that player A wins any particular rally against player B. Although $\phi_3(\theta)$ is a piecewise function, consideration of the obvious points is sufficient to prove that it is continuous.

Now suppose that player A opts to set to three points when the score reaches 9-9 with player B serving. Then she can win the game according to any of the following ten possible scoring patterns.

(0-0) $\rightarrow$ (1-0) $\rightarrow$ (2-0) $\rightarrow$ (3-0)
(0-0) $\rightarrow$ (1-0) $\rightarrow$ (2-0) $\rightarrow$ (2-1) $\rightarrow$ (3-1)
(0-0) $\rightarrow$ (1-0) $\rightarrow$ (1-1) $\rightarrow$ (2-1) $\rightarrow$ (3-1)
(0-0) $\rightarrow$ (0-1) $\rightarrow$ (1-1) $\rightarrow$ (2-1) $\rightarrow$ (3-1)
(0-0) $\rightarrow$ (1-0) $\rightarrow$ (2-0) $\rightarrow$ (2-1) $\rightarrow$ (2-2) $\rightarrow$ (3,2)
(0-0) $\rightarrow$ (1-0) $\rightarrow$ (1-1) $\rightarrow$ (2-1) $\rightarrow$ (2-2) $\rightarrow$ (3,2)
(0-0) $\rightarrow$ (1-0) $\rightarrow$ (1-1) $\rightarrow$ (1-2) $\rightarrow$ (2-2) $\rightarrow$ (3,2)
(0-0) $\rightarrow$ (0-1) $\rightarrow$ (1-1) $\rightarrow$ (2-1) $\rightarrow$ (2-2) $\rightarrow$ (3,2)
(0-0) $\rightarrow$ (0-1) $\rightarrow$ (1-1) $\rightarrow$ (1-2) $\rightarrow$ (2-2) $\rightarrow$ (3,2)
(0-0) $\rightarrow$ (0-1) $\rightarrow$ (0-2) $\rightarrow$ (1-2) $\rightarrow$ (2-2) $\rightarrow$ (3,2)

Defining $\phi_4(\theta)$ to be the probability that player A wins the endgame from 9-9 if she opts to set to three points, we thus have

$$\begin{aligned} \phi_4(\theta) = {} & P(A|B)P(A|A)P(A|A) + P(A|B)P(A|A)P(B|A)P(A|B) + \\ & P(A|B)P(B|A)P(A|B)P(A|A) + P(B|B)P(A|B)P(A|A)P(A|A) + \\ & P(A|B)P(A|A)P(B|A)P(B|B)P(A|B) + P(A|B)P(B|A)P(A|B)P(B|A)P(A|B) + \\ & P(A|B)P(B|A)P(B|B)P(A|B)P(A|A) + P(B|B)P(A|B)P(A|A)P(B|A)P(A|B) + \\ & P(B|B)P(A|B)P(B|A)P(A|B)P(A|A) + P(B|B)P(B|B)P(A|B)P(A|A)P(A|A) \\ = {} & \frac{\theta^4\left(\theta^2 - 3\theta + 3\right)\left(\theta^4 + \theta^3 - 3\theta^2 + \theta + 1\right)}{\left(\theta^2 - \theta + 1\right)^5} \end{aligned} \tag{15}$$

from Equations (1) to (4) after some algebraic manipulation.

Figure 3 illustrates graphs of $\phi_3(\theta)$ and $\phi_4(\theta)$, the probabilities of winning when playing through to eleven points and when setting to three points respectively, in women's singles games under the old rules. As for Equation (7), when the option of setting arises at 10-10, we need to determine the values of θ for which $\phi_3(\theta) = \phi_4(\theta)$. Due to the piecewise nature of $\phi_3(\theta)$, we must solve this equation in each of the three relevant intervals. Firstly for $0 < \theta < 0\cdot 382$, Equations (11) and (15) give

$$\phi_{3(1)}(\theta)-\phi_4(\theta)=-\frac{\theta^3(1-\theta)^2(\theta^2-\theta-1)(\theta^2-3\theta+1)}{(\theta^2-\theta+1)^5}, \tag{16}$$

which has zeroes at 0, 1, $(1\pm\sqrt{5})/2$ and $(3\pm\sqrt{5})/2$. This list includes both endpoints of the feasible interval but no values within it. As $\phi_{3(1)}(\theta)>\phi_4(\theta)$ in this interval, player A should not set to three at 9-9 if $0<\theta<0\cdot 382$. Secondly for $0\cdot 382<\theta<0\cdot 618$, Equations (12) and (15) give

$$\phi_{3(2)}(\theta)-\phi_4(\theta)=\frac{\theta^3(1-\theta)^2(\theta^2-3\theta+1)}{(\theta^2-\theta+1)^5}, \tag{17}$$

which has zeroes at 0, 1 and $(3\pm\sqrt{5})/2$. This list includes only the lower endpoint of the feasible interval and no values within it. As $\phi_{3(2)}(\theta)<\phi_4(\theta)$ in this interval, player A should set to three at 9-9 if $0\cdot 382<\theta<0\cdot 618$. Thirdly for $0\cdot 618<\theta<1$, Equations (13) and (15) give

$$\phi_{3(3)}(\theta)-\phi_4(\theta)=\frac{\theta^3(1-\theta)^2(\theta^3+\theta^2-5\theta+2)}{(\theta^2-\theta+1)^5}, \tag{18}$$

which has zeroes at 0, 1, $-2\cdot 935$, $0\cdot 463$ and $1\cdot 473$ to three decimal places. This list includes only the upper endpoint of the feasible interval and no values within it. As $\phi_{3(3)}(\theta)<\phi_4(\theta)$ in this interval, player A should set to three at 9-9 if $0\cdot 618<\theta<1$.

Although the probability that player A wins the game is less affected by whether she sets at 9-9 that it is at 10-10, as evident by visual inspection of the graphs in Figures 2 and 3, amazingly the decision threshold is the same. That is, player A should play through at 9-9 and 10-10 if $\theta<0\cdot 382$ but should opt to set instead if $\theta>0\cdot 382$. We now ask whether this is mere coincidence and what the setting decision thresholds are for events other than women's singles.

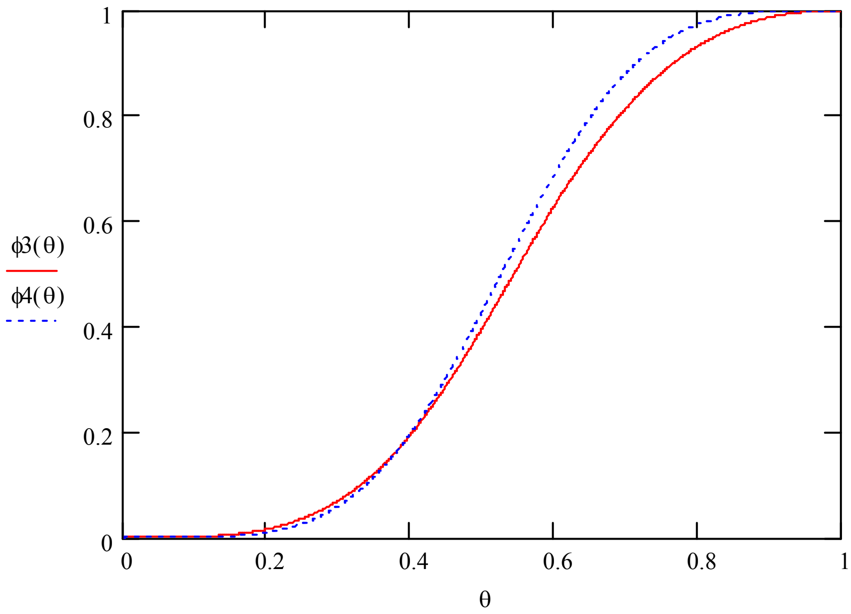

Figure 3: probabilities of winning when playing through, $\phi_3(\theta)$, and setting to three points, $\phi_4(\theta)$, at 9-9 in women's singles games for different values of θ.

2.2. Men's Singles Events

The analytical and numerical calculations developed for women's singles in Subsection 2.1 extend naturally to men's singles, for which setting options arise under the old rules at scores of 13-13 (setting to five points) and 14-14 (setting to three points) in games to fifteen points.

2.2.1 Men's Singles 14-14

At 14-14 with player B serving, the probability that player A wins the game if he opts to play through is

$$\phi_5(\theta) = P(A|B) = \frac{\theta^2}{\theta^2 - \theta + 1} \tag{19}$$

from Equation (1). If he opts to set to three instead, the probability that player A wins the game is

$$\phi_6(\theta) = \frac{\theta^4(\theta^2 - 3\theta + 3)(\theta^4 + \theta^3 - 3\theta^2 + \theta + 1)}{(\theta^2 - \theta + 1)^5} \tag{20}$$

from Equation (15).

To determine what values of θ lead to recommendations of playing through and setting to three points at 14-14 in men's singles games, we investigate the conditions under which $\phi_5(\theta) = \phi_6(\theta)$. Consider the difference

$$\phi_5(\theta)-\phi_6(\theta)=\frac{\theta^2(1-\theta)(2\theta^6-11\theta^5+18\theta^4-12\theta^3+4\theta^2-3\theta+1)}{(\theta^2-\theta+1)^5}, \tag{21}$$

which has zeroes at 0, 1, $0\cdot413$, $3\cdot256$, $-0\cdot169\pm0\cdot486i$ and $1\cdot085\pm0\cdot478i$ to three decimal places. A little further calculation reveals that the recommended strategy in men's singles at 14-14 is that player A should play through to fifteen points if $\theta<0\cdot413$ and should set to three points if $\theta>0\cdot413$, as illustrated in Figure 4. The greatest difference in the interval $0<\theta<1$ occurs at $\theta\approx0\cdot70$, where $|\phi_5(\theta)-\phi_6(\theta)|\approx0\cdot26$, so the probability of winning a game can increase by one quarter if a player makes the correct decision.

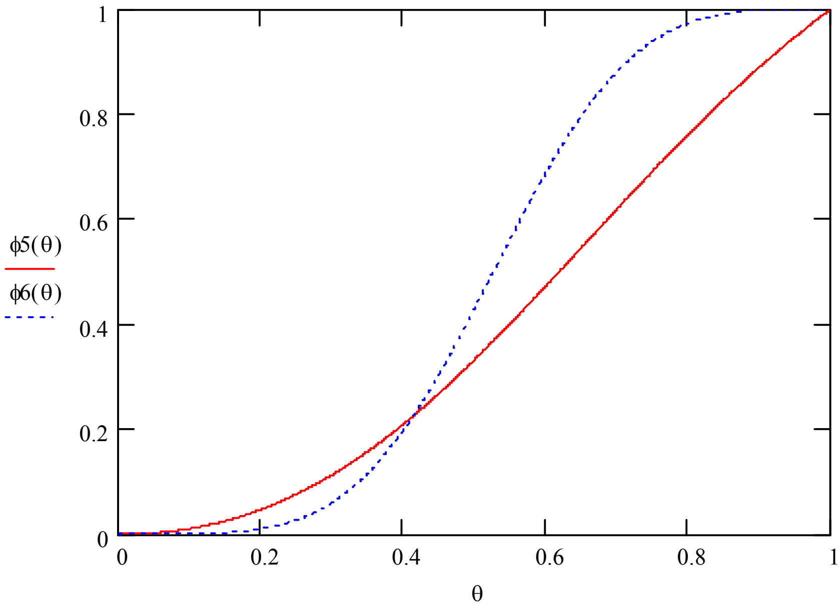

Figure 4: probabilities of winning when playing through, $\phi_5(\theta)$, and setting to three points, $\phi_6(\theta)$, at 14-14 in men's singles games for different values of θ.

2.2.2 Men's Singles 13-13

The corresponding algebra for setting decisions at 13-13 is very protracted, so we merely summarize the results here. As for women's singles at 9-9, we need to take account of possible setting at 14-14 if a player opts to play through at 13-13. Firstly in the interval $0<\theta<0\cdot413$, the probability that player A wins the endgame if he opts to play through at 13-13 is

$$\phi_{7(1)}(\theta)=\frac{\theta^3}{(\theta^2-\theta+1)^7}\left(\begin{array}{l}\theta^{11}-5\theta^{10}+11\theta^9-13\theta^8+15\theta^7-25\theta^6\\+29\theta^5-11\theta^4-8\theta^3+10\theta^2-4\theta+1\end{array}\right). \tag{22}$$

Secondly in the interval $0\cdot413<\theta<0\cdot587$, the probability that player A wins the endgame if he opts to play through at 13-13 is

$$\phi_{7(2)}(\theta)=\frac{\theta^3}{\left(\theta^2-\theta+1\right)^7}\begin{pmatrix}\theta^{11}-3\theta^{10}-6\theta^9+44\theta^8-86\theta^7+80\theta^6\\-40\theta^5+23\theta^4-24\theta^3+16\theta^2-5\theta+1\end{pmatrix}. \tag{23}$$

Thirdly in the interval $0\cdot587<\theta<1$, the probability that player A wins the endgame if he opts to play through at 13-13 is

$$\phi_{7(3)}(\theta)=\frac{\theta^3}{\left(\theta^2-\theta+1\right)^7}\begin{pmatrix}\theta^{11}-3\theta^{10}-4\theta^9+37\theta^8-84\theta^7+106\theta^6\\-94\theta^5+74\theta^4-53\theta^3+29\theta^2-10\theta+2\end{pmatrix}. \tag{24}$$

Collating these results gives the probability that player A wins the endgame from 13-13 if he opts to play through to fifteen points as

$$\phi_7(\theta)=\begin{cases}\phi_{7(1)}(\theta); & 0<\theta<0\cdot413\\ \phi_{7(2)}(\theta); & 0\cdot413<\theta<0\cdot587\\ \phi_{7(3)}(\theta); & 0\cdot587<\theta<1\end{cases} \tag{25}$$

in terms of θ, the assumed constant probability that player A wins any particular rally against player B. Although $\phi_7(\theta)$ is a piecewise function, consideration of the obvious points is sufficient to prove that it is continuous.

Now suppose that player A opts to set to five points when the score reaches 13-13 with player B serving. After a great deal of effort, we can calculate the probability that player A wins the game as

$$\phi_8(\theta)=\frac{\theta^6}{\left(\theta^2-\theta+1\right)^9}\begin{pmatrix}\theta^{12}-4\theta^{11}-10\theta^{10}+94\theta^9-211\theta^8+119\theta^7+265\theta^6\\-504\theta^5+276\theta^4+50\theta^3-100\theta^2+20\theta+5\end{pmatrix}. \tag{26}$$

To determine what values of θ lead to recommendations of playing through and setting to five points at 13-13 in men's singles games, we investigate the conditions under which $\phi_7(\theta)=\phi_8(\theta)$. By considering the piecewise differences of these functions, it becomes clear that player A should play through to fifteen points if $\theta<0\cdot440$ and should set to five points if $\theta>0\cdot440$ as illustrated in Figure 5, which presents graphs of the corresponding probabilities of winning the game.

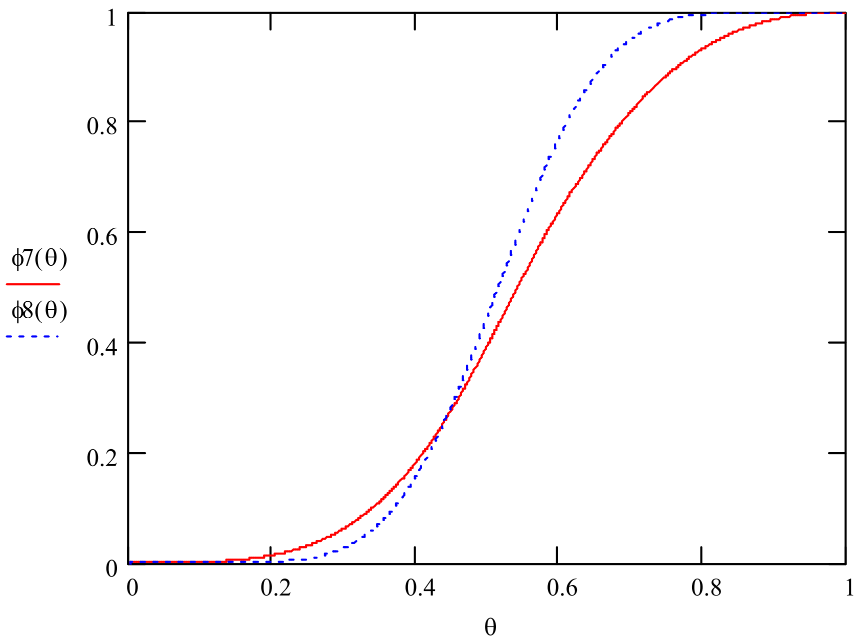

Figure 5: probabilities of winning when playing through, $\phi_7(\theta)$, and setting to three points, $\phi_8(\theta)$, at 13-13 in men's singles games for different values of θ.

2.3. All Doubles Events

The setting options for all doubles events under the old rules are the same as for men's singles: to five points at 13-13 and to three points at 14-14 in games to fifteen points. However, new terminology is required to allow for the fact that doubles players in a team generally both serve during any turn. By defining and calculating the probability that team A wins the next point at first server given that team A is currently at first server as

$$P(A_1|A_1) = \theta\sum_{i=0}^{\infty}\left\{\theta^2(1-\theta)^2\right\}^i = \frac{\theta}{1-\theta^2+2\theta^3-\theta^4}, \tag{27}$$

the probability that team A wins the next point at first server given that team A is currently at second server as

$$P(A_1|A_2) = (1-\theta)\theta^3\sum_{i=0}^{\infty}\left\{\theta^2(1-\theta)^2\right\}^i = \frac{\theta^3(1-\theta)}{1-\theta^2+2\theta^3-\theta^4}, \tag{28}$$

the probability that team A wins the next point at second server given that team A is currently at first server as

$$P(A_2|A_1) = (1-\theta)\theta\sum_{i=0}^{\infty}\left\{\theta^2(1-\theta)^2\right\}^i = \frac{\theta(1-\theta)}{1-\theta^2+2\theta^3-\theta^4} \tag{29}$$

and so on, we can repeat the analyses of the singles events. For example, if team A opts to play through at 14-14 when team B is first server, the probability that team A wins the game

is $P(A_1|B_1)+P(A_2|B_1)=\theta^3(2-\theta)/(1-\theta^2+2\theta^3-\theta^4)$. However, the algebra involved is extremely cumbersome, particularly when the score is 13-13 or team A opts to set, and so we determine the doubles events decision thresholds using simulation in Fortran.

Table 2 summarizes the setting decision thresholds for all events under the old rules. Notice that the thresholds in doubles events are less if the team serving is at first server, than they are if it is at second server. For all events, our assumption that team A has a constant probability θ of winning any rally against team B and that the outcomes of all rallies are independent provides a reasonable model. Any relaxation of this assumption would likely make the whole analysis infeasible in practice.

Event	Score	Setting interval
women's singles	10-10	$\theta > 0\cdot382$
women's singles	9-9	$\theta > 0\cdot382$
men's singles	14-14	$\theta > 0\cdot413$
men's singles	13-13	$\theta > 0\cdot440$
all doubles	14-14 (first server)	$\theta > 0\cdot318$
all doubles	14-14 (second server)	$\theta > 0\cdot417$
all doubles	13-13 (first server)	$\theta > 0\cdot372$
all doubles	13-13 (second server)	$\theta > 0\cdot469$

Table 3: optimal setting decision thresholds under the old rules, where θ is the probability that the team with the option wins any particular rally.

3. Probabilities of Winning

Having established optimal endgame strategies under the old rules, we now incorporate these decision thresholds into calculations of probabilities that particular players or teams win complete games against particular opponents. This enables us to compare the fairness and discriminatory ability of the new rules with those of the old rules, in order to judge their respective merits. Under the old rules, players and teams can only score when serving and different calculations are needed depending on which player or team serves first. Consequently, the algebra to determine probabilities of winning games is considerably more complicated than that in Section 2 and is a pointless exercise when computer simulation is relatively easy to program, quick to run and produces accurate results. We use the language Fortran for this purpose.

However, the corresponding algebra for the new rules is far easier because players or teams score points from all rallies. We present these details here to enable further understanding of the process and because it serves as a valuable check that the programming code is correct. In passing, the formulae below use the binomial probability that the score in a game reaches 20-20, so invoking the endgame rule. This probability is equal to

$$\binom{40}{20}\theta^{20}(1-\theta)^{20}$$

and it is interesting to note that the probability is about one in eight if $\theta = 0\cdot 5$ but is only about one in two thousand five hundred if $\theta = 0\cdot 25$ or $\theta = 0\cdot 75$. It is also interesting to note that the above binomial coefficient implies that there are about $1\cdot 4\times 10^{11}$ different scoring patterns that would lead to a game score of 20-20.

The first observation to make is that for all events under the new rules, team A can win by any of the scores 21-0, 21-1, …, 21-19 with corresponding probability of winning the game equal to

$$\sum_{i=0}^{19}\binom{20+i}{20}\theta^{21}(1-\theta)^{i} ,$$

any of the scores 22-20, 23-21, …, 30-28 with corresponding probability of winning the game equal to

$$\left\{\binom{40}{20}\theta^{20}(1-\theta)^{20}\right\}\left\{\sum_{i=0}^{8}2^{i}\theta^{2+i}(1-\theta)^{i}\right\},$$

or the score 30-29 with corresponding probability of winning the game equal to

$$\left\{\binom{40}{20}\theta^{20}(1-\theta)^{20}\right\}\left\{2^{9}\theta^{10}(1-\theta)^{9}\right\}.$$

Adding these three terms together gives the probability that team A wins a game against team B under the new rules, whichever team serves first.

3.1. Probabilities of Winning a Game

Figure 6 illustrates graphs of the probabilities that player A wins a game against player B under the old rules, when serving first and second respectively, and under the new rules in women's singles events. There are several features to note, not least that the curve for the new rules is the same regardless of who serves first. As the initial server for the first game in a match is decided randomly, the new rules appear to be fairer in this regard. However, we comment more on this point later. Most importantly, there is very close agreement in the probability curves under the old and new rules. This implies that both sets of rules are equally able to discriminate between the better and poorer player in women's singles events. However, we also note that the probabilities for the old rules are slightly greater when serving first, and slightly less when serving second, compared with those for the new rules.

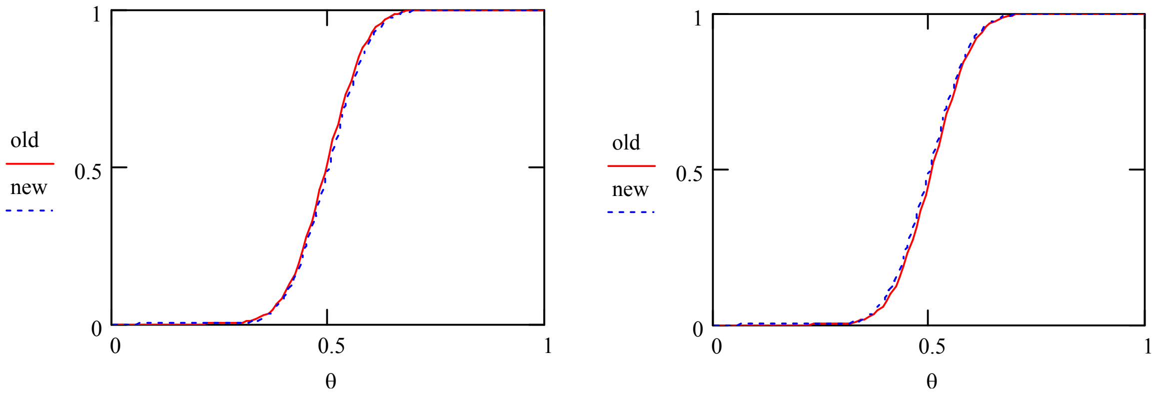

Figure 6: probabilities of winning a women's singles game for different values of θ under the old and new rules, when serving first and second respectively.

Similarly, Figure 7 illustrates graphs of the probabilities that player A wins a game against player B under the old rules, when serving first and second respectively, and under the new rules in men's singles events. Again, the curve for the new rules is the same regardless of who serves first and there is close agreement in the probability curves under the old and new rules. This implies that both sets of rules are similarly able to discriminate between the better and poorer player in men's singles events, though the old rules are clearly better in this regard. The influence of serving first or second under the old rules has largely disappeared, as men play to fifteen points whereas women only play to eleven points.

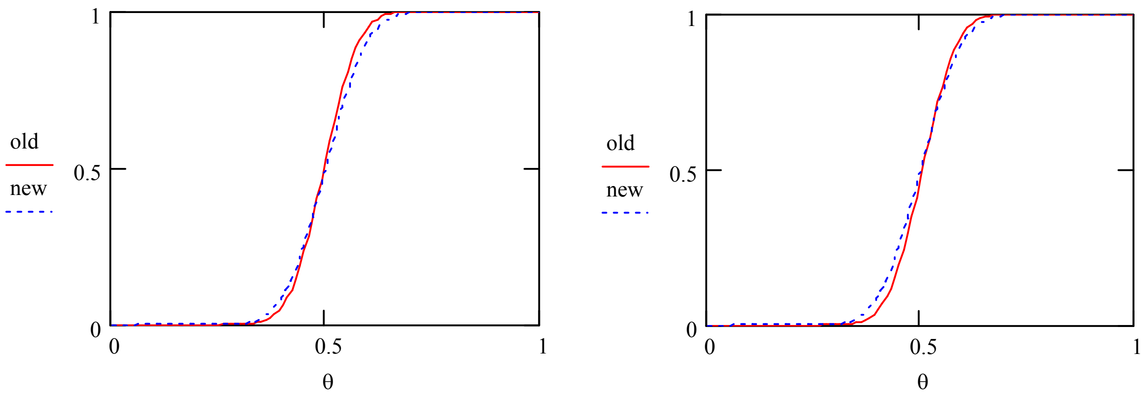

Figure 7: probabilities of winning a men's singles game for different values of θ under the old and new rules, when serving first and second respectively.

Finally, Figure 8 illustrates graphs of the probabilities that team A wins a game against team B under the old rules, when serving first and second respectively, and under the new rules in all doubles events. The curves are very similar to those for the men's singles events and the same comments apply.

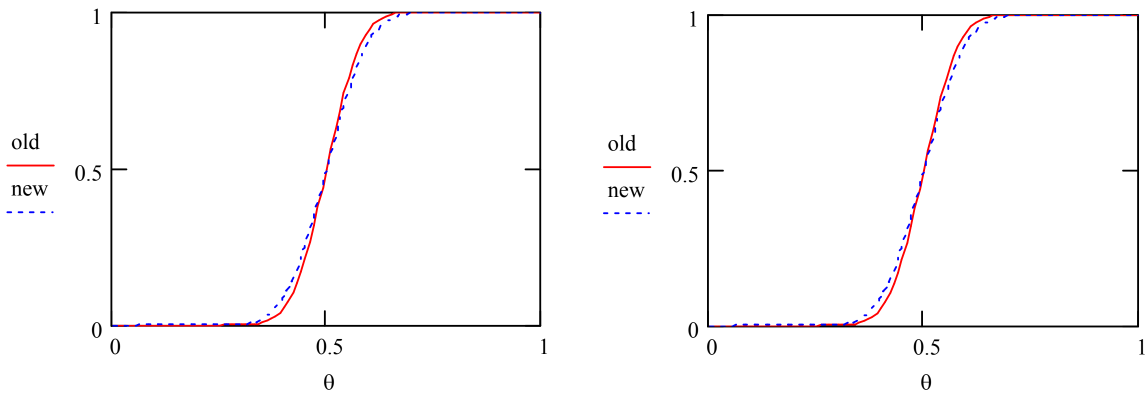

Figure 8: probabilities of winning a doubles game for different values of θ under the old and new rules, when serving first and second respectively.

3.2. Probabilities of Winning a Match

Suppose that team A has probability ϕ of winning any game against team B, according to our calculations in Subsection 3.1. Under the new rules, this probability does not depend on who serves and so the probability that team A wins a match against team B is given by

$$\psi = \phi^2 + \phi(1-\phi)\phi + (1-\phi)\phi^2 = \phi^2(3-2\phi). \tag{30}$$

We illustrate the probabilities of winning a game and a match under the new rules in Figure 9, in terms of the constant probability θ that team A wins any rally against team B. Clearly, by playing a match to the best of three games, we achieve greater discriminatory ability than by playing for just one game. This means that the better player will win on the vast majority of occasions. However, one could debate whether this is fair or whether a player with probability θ of winning a rally should perhaps win a proportion θ of matches on average.

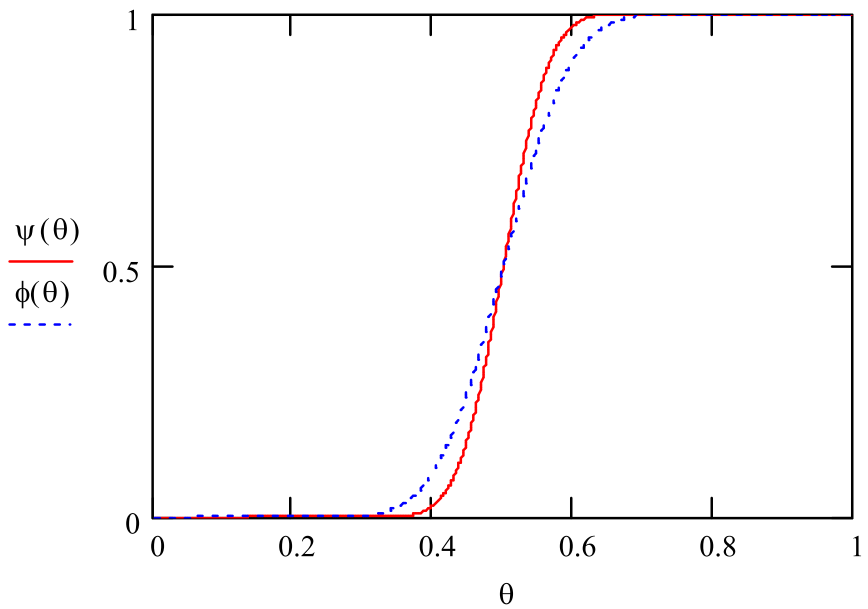

Figure 9: probabilities of winning a game $\phi(\theta)$ and a match $\psi(\theta)$ for different values of θ under the new rules.

Similar comparisons between the probabilities of winning a game and a match are possible for the old rules. However, we need some adjustments in this case because the initial server in each game affects the probability of winning that game and because each event has its own probability curve as presented in Subsection 3.1. The initial server in the first game is decided at random and the initial server in any subsequent game is the winner of the previous game. We do not present this analysis here, as it would not add any new substance to this article.

4. Entertainment Value

The IBF's main aspirations by introducing the new rules were to make the game faster and more entertaining. Without changing any rules other than scoring rules, we could make the game faster by shortening the duration of rallies, games or matches. Under the old rules, but not the new rules, a team cannot lose a point when serving and so might attempt riskier shots. This might make for an exciting game with shorter rallies, though we do not investigate this aspect here. Both set of rules involve matches comprising the best of three games and so we only need compare the durations of games under the two systems.

To do this, we make the simplifying assumption that the duration of a game is directly proportional to the number of rallies in that game. This supposes that the average length of a rally is the same for all events under both sets of rules. We again assume that there is a constant probability that a specified team wins any particular rally and that outcomes of rallies are independent. By simulation, we evaluate the mean

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} r_i \tag{31}$$

and approximate 95% probability interval with limits $\hat{\mu} \pm 1\cdot 96\hat{\sigma}$, where the standard deviation is defined by

$$\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}\left(r_i - \hat{\mu}\right)^2 \tag{32}$$

and r_i is the number of rallies in game $i = 1,2,\ldots,n$. For our calculations, declaring $n = 10^6$ proves ample to achieve algorithmic convergence and accurate estimation. Figure 10 presents the means and approximate 95% probability intervals under the new rules, for which the minimum number of rallies is twenty-one and the maximum is fifty-nine. Although smaller minima of eleven and fifteen rallies exist for events under the old rules, there are no maxima and long games are possible.

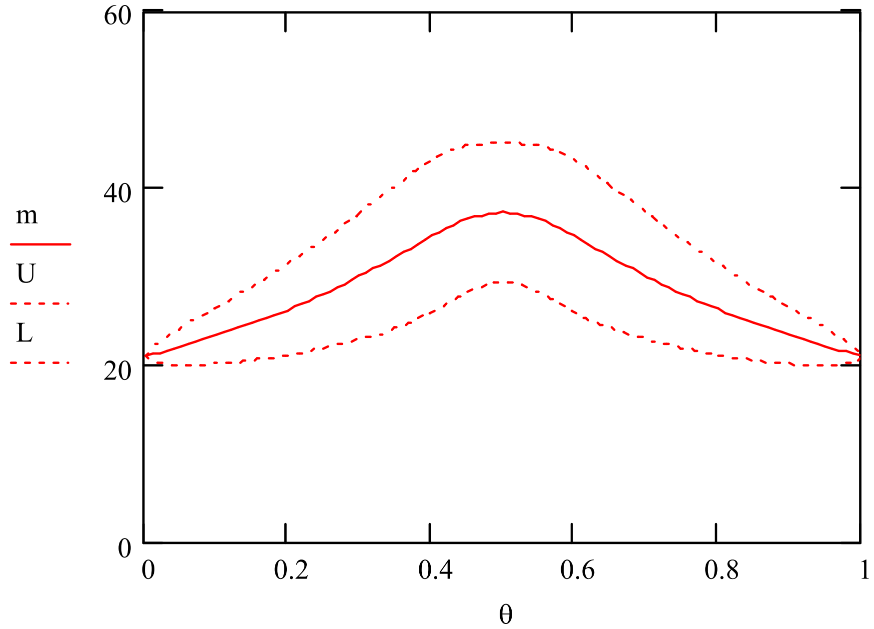

Figure 10: mean (m) numbers of rallies per game and approximate 95% probability intervals (L,U) for different values of θ under the new rules.

Figure 11 presents the means for the new rules and for the old rules, separately for women's singles, men's singles and all doubles events in the latter case. Which player or team serves first affects the means slightly: the graphs in Figure 11 all correspond to the case where player or team A serves first. On average, there are fewer rallies per women's singles game under the old rules, than there are under the new rules, for all θ. The same is true for all other events if θ or $1-\theta$ is less than about one quarter. However, the mean number of rallies per game is greater under the old rules, than it is under the new rules, for values of θ that are closer to one half. As mentioned above though, there is an upper limit to the number of rallies per game in the new rules, whereas there is not under the old rules.

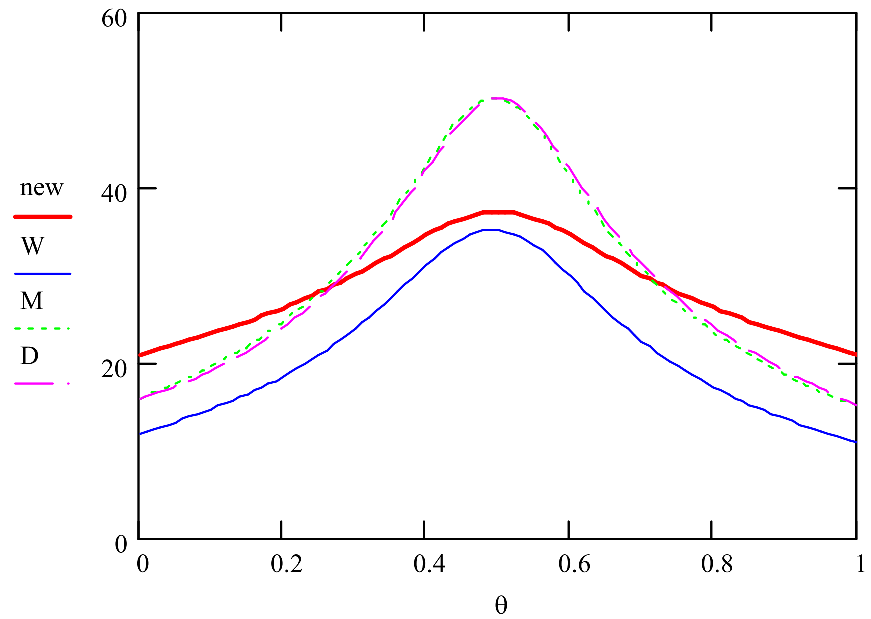

Figure 11: mean numbers of rallies per game for different values of θ corresponding to women's (W) and men's (M) singles and all doubles (D) events under the old rules.

We now switch our attention from how fast a game is to how entertaining it is. This is a subjective description, which means different things to different people. Possible features of exciting competition are that the outcome is uncertain, matches, games and rallies are neither too short nor too long and there are continual changes to the score. Section 3 showed that the outcomes of games and matches are comparable, and we have just shown that games and matches have similar durations, under both sets of rules, except that the new rules avoid lengthy games that occasionally arise under old rules. Of course, the very rarity of such events might make them particularly exciting to players and spectators. However, the new rules do ensure continual changes to the score after every rally, whereas the old rules often involve scores remaining unchanged for several rallies.

5. Commonwealth Games

We now investigate how the new rules performed at the first competition after their implementation, the 2006 Commonwealth Games in Melbourne, Australia. Table 4 presents the results of the badminton finals at this competition; see the Commonwealth Games website (2006). None of the events went to three games, though Section 3 suggests that this would have been the case under the old rules too. From these match scores, we can calculate naïve estimates of θ, as the proportion of rallies actually won, such as $42/69$ for the women's singles. In doing so, we take the eventual winners as team A and the runners-up as team B without loss of generality. This estimate is the sample proportion $\hat{\theta}$ based on the observed data, which is also the maximum likelihood estimate of θ, though we would not be interested in predicting match outcome if these data were already observed. We display these estimates in the last column of Table 4.

Event	Players	Game 1	Game 2	Game 3	$\hat{\theta}$
women's singles	Hallam (ENG) Wong (MAS)	21 12	21 15	–	$0\cdot61$
men's singles	Lee (MAS) Wong (MAS)	21 13	21 12	–	$0\cdot63$
mixed doubles	Emms & Robertson (ENG) Runesten Petersen & Shirley (NZL)	21 17	21 10	–	$0\cdot61$
women's doubles	Chin & Wong (MAS) Jiang & Li (SIN)	21 17	21 19	–	$0\cdot54$
men's doubles	Chan & Koo (MAS) Choong & Wong (MAS)	21 13	21 14	–	$0\cdot61$

Table 4: results of badminton finals at the 2006 Commonwealth Games.

For these estimates, we now calculate three quantities under each of the old and new rules. They are the probability ϕ that team A wins any particular game against team B, the mean number m of rallies per game and an approximate 95% probability interval (L,U) for the number of rallies per game. Table 5 presents this information for easy reference. The probabilities of winning a game are very similar under both scoring systems, which is

somewhat reassuring. The mean numbers of rallies per game under the new rules agree very closely with the actual numbers of rallies observed. The mean would have been less under the old rules for the women's singles event, but would have been several rallies greater for all other events. Finally, the upper probability limits for all events are considerably less for the new rules than for the old rules. These observations support our earlier findings and confirm that the results of these finals matches appear to have been successful.

	Old rules			New rules		
Event	ϕ	m	(L,U)	ϕ	m	(L,U)
women's singles	$0\cdot94$	29	(11,47)	$0\cdot93$	34	(26,42)
men's singles	$0\cdot98$	38	(18,58)	$0\cdot96$	33	(25,41)
mixed doubles	$0\cdot96$	41	(19,63)	$0\cdot93$	34	(26,42)
women's doubles	$0\cdot73$	48	(24,72)	$0\cdot69$	37	(29,45)
men's doubles	$0\cdot96$	41	(19,63)	$0\cdot93$	34	(26,42)

Table 5: comparisons of scoring rules for the 2006 Commonwealth Games.

6. Predicting Match Outcomes

The ability to predict match outcomes is useful for determining handicaps or seeding for tournaments and for gambling purposes. In both cases, we need to predict the result before a match begins, though there is potential in the latter case for updating odds calculations as a match progresses. In Section 5, we estimated the probability θ that team A wins a rally against team B based upon the subsequent result of a match. This clearly is unsuitable in addressing the above requirements. We could calculate an alternative estimate from historical data relating to previous encounters between these two teams, though this is likely to be inaccurate as long-term and short-term variability in performance often dominates.

A Bayesian analysis is ideally suited to this scenario, as it enables us to combine subjective judgements and historical data to express our knowledge about the unknown parameter θ in terms of a prior distribution with probability density function $g(\theta)$. Moreover, we can update this prior distribution continually during play, into an evolving posterior distribution. As shown by Bernardo and Smith (1993), for example, the natural conjugate prior for this scenario is the beta density

$$g(\theta)=\frac{1}{\mathrm{B}(a,b)}\theta^{a-1}(1-\theta)^{b-1}\;;\quad 0<\theta<1 \tag{33}$$

in terms of hyperparameters a and b that are chosen to reflect our prior knowledge accurately. Percy (2003) described and illustrated a suitable method for eliciting these hyperparameters.

Before a match starts, we can take expectations over this prior distribution in order to allow for the uncertainty in θ. For example, Section 2 evaluates the probability $\phi(\theta)$ that team A wins a game against team B. Rather than substitute an inaccurate estimate for θ into this function, we average over the prior to give

$$\phi = E_\theta\{\phi(\theta)\} = \int_0^1 \phi(\theta)g(\theta)d\theta . \tag{34}$$

During the course of a game, the outcomes of actual rallies will become available and so we will be able to evaluate an evolving binomial likelihood of the form

$$L(\theta; A, B) \propto \theta^A (1-\theta)^B \tag{35}$$

where A is the number of rallies won by team A and B is the number of rallies won by team B. We can then replace the prior in Equation (34) by the posterior

$$g(\theta|A, B) \propto L(\theta; A, B)g(\theta), \tag{36}$$

which also has a beta form

$$g(\theta|A, B) = \frac{1}{\mathrm{B}(a + A, b + B)}\theta^{a+A-1}(1-\theta)^{b+B-1} \ ; \quad 0 < \theta < 1 \tag{37}$$

and so gives

$$\phi = E_{\theta|A,B}\{\phi(\theta|A, B)\} = \int_0^1 \phi(\theta|A, B)g(\theta|A, B)d\theta . \tag{38}$$

Observe that we have replaced the probability $\phi(\theta)$ of winning a game before playing any rallies, given the known parameter θ, by a new function $\phi(\theta|A, B)$ that reflects how this probability has changed due to rallies already played in the game. Analytical evaluation of these probabilities might be possible for setting decisions under the old rules and for predicting winners under the new rules before commencement of play. However, we need numerical integration for predicting winners under the old rules and for predicting winners during play for both sets of rules, so we might as well employ quadrature in all cases when we adopt a Bayesian approach.

6.1. Example

We illustrate the Bayesian approach with a simple example. Suppose I believe that the proportion of rallies θ that I would win against a particular opponent is equally likely to lie in the intervals $(0\cdot 0, 0\cdot 6)$, $(0\cdot 6, 0\cdot 7)$ and $(0\cdot 7, 1\cdot 0)$. This provides two constraints for the subjective prior distribution of the forms

$$G(0\cdot 6) = \frac{1}{3} \quad \text{and} \quad G(0\cdot 7) = \frac{2}{3} \tag{39}$$

where

$$G(\theta) = \int_{-\infty}^{\theta} g(\theta)d\theta \tag{40}$$

is the prior cumulative distribution function, corresponding to a beta distribution here from Equation (33). We now solve the pair of simultaneous nonlinear Equations (39) numerically to determine values for the hyperparameters a and b. From Mathcad software, we obtain $a \approx 11 \cdot 1$ and $b \approx 6 \cdot 1$. In this way, we elicit expert subjective knowledge about θ and convert it into a completely specified prior distribution, with probability density function given by Equation (33) and illustrated in Figure 12.

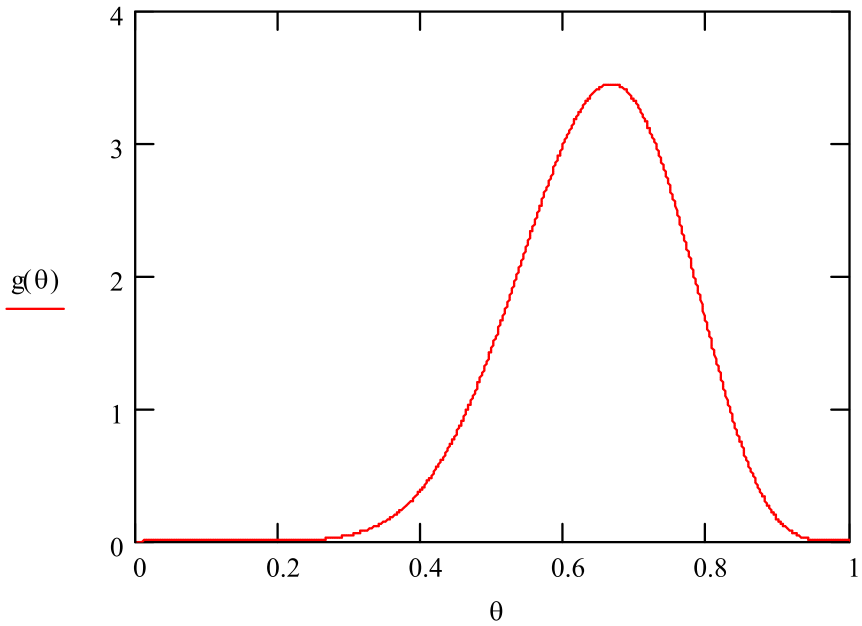

Figure 12: example of subjective prior probability density function for θ.

Suppose the score in my men's singles game under the old rules reaches 14-14 with my opponent serving and I wish to calculate my probability of winning the game if I opt to play through to fifteen points. From Equations (19) and (34), we average $\phi_5(\theta)$ over the unknown parameter's prior distribution to give

$$\phi_5 = E_\theta\{\phi_5(\theta)\} = \int_0^1 \frac{\theta^2}{\theta^2 - \theta + 1} g(\theta)d\theta = 0 \cdot 54. \tag{42}$$

to two decimal places using numerical quadrature in Mathcad. Had we naïvely set $\theta = 0 \cdot 65$ instead of allowing for its uncertainty by means of a prior distribution, we would have calculated the excellent approximation $\phi_5(\theta) \approx 0 \cdot 55$. However, this approximation might be poor on occasions and we cannot readily supplement it with observed data, unlike the prior-posterior updating procedure described above. Moreover, $\phi_5(\theta)$ is one of the simplest objective functions that we encountered in this research – others are likely to result in poor approximations unless prior distributions are included in the analyses.

7. Conclusions

This article began by reviewing recent changes to the scoring rules introduced by the International Badminton Federation to make the game faster and more entertaining. By applying combinatorial, probabilistic and simulation methods to extrapolate known probabilities of winning individual rallies into probabilities of winning games and then matches, we demonstrated that the overall fairness and discriminatory ability of the new scoring system are similar to those of the old scoring system for all events. The new rules are fairer inasmuch as they apply uniformly to all events, whereas the old rules had a different scoring scheme for the women's singles event.

Our analyses also measured how well the rule changes meet the IBF's aspirations by comparing the numbers of rallies per game and the scoring patterns within each game under both sets of rules. We conclude that the new rules lead to faster and more exciting games in this regard, though we note that entertainment value is a subjective term that we could measure in many ways. Counteracting these benefits is the negative effect that the new rules lose the tradition and uniqueness of the old rules. We then demonstrated our results using actual performance data from the 2006 Commonwealth Games to assess and compare the two scoring systems, confirming that all the right players won.

Finally, we resolved the difficulties of parameter estimation by developing subjective Bayesian methods for specifying the probabilities of winning individual rallies and described how to propagate this information with observed data to determine posterior predictive distributions for match outcomes, so enabling us to predict match outcomes for given scenarios before and during play. The techniques thus developed could prove very useful for evaluating new proposed scoring rules in badminton and other sports, without having to put them in practice first, thus avoiding unfortunate situations such as that which occurred when three sets of experimental badminton rules were implemented in 2002.

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