The Importance of a Match in a Tournament

Philip Scarf and Xin Shi

Centre for Operational Research and Applied Statistics,
Salford Business School,
University of Salford,
Salford,
Manchester
M5 4WT, UK.

email p.a.scarf@salford.ac.uk

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Philip Scarf[1] and Xin Shi

Centre for Operational Research and Applied Statistics, Salford Business School, University of Salford, Salford, Manchester M5 4WT, UK.

Abstract

A quantitative measure of "match importance" is useful in a number of decision problems, for example: to inform policy-making regarding income distribution in sport; as a metric in tournament design; for selecting matches for broadcasting; for scheduling matches in a tournament; for assigning referees; and for determining when competitors should make the greatest effort. To date measures of match importance used in such analyses have been relatively naïve. We discuss a general measure that considers the effect of a particular match on the end of tournament position, given the results of all other matches, some played, some predicted. We use logistic regression to predict matches and Monte Carlo simulation to compute the match importance measure, and apply these to soccer matches in the English Football Association Premier League.

Keywords: sport; contest; outcome uncertainty; championship significance; soccer.

1. Introduction

In sport, taking part is important, but success matters—competitors play to win, spectators want to watch winners. Therefore, in a sports tournament in which teams or competitors play ties or individual matches, the progression or elimination of teams or individual competitors is fundamental for maintaining the interest of participants and spectators. Szymanski [1] has also argued that competitors, particularly in American team sports, are profit maximisers and hence play for profit. The financial incentive for winning is high. The incentive for surviving will also be significant, since relegation from a leading league division, for example, will have serious consequences for future income from broadcasting contracts in particular. Thus, at both ends of the performance spectrum, winning and surviving matter. Competitors are expected to raise their game in matches that are critical, and Economists (e.g. Jennett [2]; Cairns [3]; Forrest *et al.* [4]) postulate that consumer demand for sport is higher when the “championship” or promotion or relegation is at stake. However, in round-robins, or the group-stages of a tournament, the outcome of particular matches may have no influence on progression or elimination (relegation) and hence the outcome of the tournament. Such unimportant matches are reduced to the status of irrelevant

[1] corresponding author: email p.a.scarf@salford.ac.uk

"friendlies" and may be form a significant proportion of the matches in a tournament. If we accept the notion that unimportant matches should be avoided, then the following question arises: how should we define or measure the importance of a match in a sports tournament? Furthermore, how might this measure of match importance be used in decision-making about, for example: the modification of tournament design; competitor effort; fixture and broadcast scheduling, and referee and player assignment?

This paper provides answers to these questions. In particular, in the following section we give a definition of match importance. Then, in section 3, the uses of such an importance measure are considered. For the definition given in section 2, a probabilistic model of match results is required for the computation of match importance—such models are described in section 4. The computation of match importance itself is discussed in section 5. The ideas of the paper are then applied in the context of the English Football Association Premier League (FAPL) in order to demonstrate how match importance can be calculated. We conclude with a discussion of remaining issues and the potential for further development.

2. Definition of match importance

Consider a round-robin in which n teams play each other in paired matches. The winner of the tournament (the "champion") is the team with the largest number of wins in individual matches after all matches have been played—this may be extended to include draws, with league-points awarded for wins and draws, and further for teams level on league-points separated by some other measure such as within-match scores (e.g. goal difference in soccer). In such a tournament and at a particular point in its play, a significant number of matches between teams with few wins thus far will have no influence on the outcome of the tournament and these matches may be considered to be unimportant. In the qualifying stages of a tournament, round-robins may be played in the first round to determine qualifiers for a subsequent second round. Again, a number of later matches in the round-robin may have no effect on determining the qualifiers. First round, round-robin designs to determine second round qualifiers are used in soccer in the FIFA World Cup and the UEFA Champions League tournaments, for example. Contrast this with a knockout, or single elimination tournament in which 2^n competitors play n elimination rounds to determine a winner, such as used in grand slam tennis tournaments. All matches have an effect on the determining the winner and are therefore important. This leads to the following working definition: a match is unimportant if it can have no influence on the tournament outcome, and match (un-)importance is a dichotomous measure here. For example, for a round-robin such as the UEFA Champions League first round group-stage (comprising four teams in each group playing a home and away round-robin with two to qualify), approximately 8% of matches have no importance (Scarf and Bilbao [5]). We might argue that only matches involving the eventual winner are important, but in reality one considers the importance of a match at the time of or before it is played—this then leads to the idea that we wish to measure the importance of a match at some time t prior to its being played.

The dichotomous measure of match importance defined above is not completely satisfactory, however, since some matches may only be important "on paper". That is, matches may be very unlikely to influence the outcome of the tournament since they may be between teams that are weak relative to those teams that are favourites to win a tournament. A significant number of

matches may fall into this category. This leads to the following definition: a match is unimportant if it is *expected* to have no influence on the tournament outcome. In this definition, we introduce the idea that the results of matches are uncertain, and so a probabilistic measure of match importance is desirable. Thus this definition supposes that *expectation* is considered with respect to the joint probability distribution of the outcomes of the matches in the tournament.

Returning to the question of time frame, we would expect a measure of importance for a particular match to change over time—for example, at the start of a season of matches in a league division tournament, say, we might consider a match scheduled for mid-season—some months away—to be important. However, by the time the match is played, the match may be unimportant as a result of the competitors being too far behind the leaders (in the league or group-stage). Thus we refine our definition further still: a match played at time $t+k$ is unimportant at time t if it is expected, at time t, to have no influence on the tournament outcome. Thus expectation here is with respect to the joint probability distribution of the outcomes of remaining matches, excepting the match of interest, given the results of matches played up to time t.

Interest in a round-robin competition is maintained not just through uncertainty about the winner, but also through other objectives for competitors such qualification for some higher competition or league. For example, in the FAPL 20 teams play a home and away round-robin with 380 matches in all over a 9-month season. Apart from contesting the championship to determine the winner of the league, the top 4 teams qualify for the UEFA Champions League—a competition for the best European teams. The 5th and 6th placed teams qualify for a minor European knockout tournament (UEFA Cup), and the bottom 3 teams are relegated to a lower division. The financial implications of these latter outcomes can be more significant than winning the championship. In this way, matches will be important (or otherwise) not just for the championship, but for *qualification* and *relegation.* Thus in general, we will need to consider the importance of a match with respect to an outcome X. Furthermore, a match may be important to one competitor but not another; at time t, competitor or team 1 may still be in contention for the championship, competitor 2 may not, and so the match 1 versus 2 at time $t+k$ will be important overall, important to 1, but not important to 2. Taking account of this, we finally arrive at our definition of match unimportance.

Definition 1: A match played at time $t+k$ is unimportant to team i at time t with respect to outcome X, if it is expected to have no influence on outcome X for team i, where expectation is with respect to the joint probability distribution of the results of remaining matches, excepting the match of interest, given the results of matches played up to time t. That is, if

$$\Pr({}_iX|{}_iW_{t+k}, H_t) - \Pr({}_iX|{}_iL_{t+k}, H_t) = 0,$$

where ${}_iX$ is the event that team i achieves outcome X (e.g. wins the championship), ${}_iW_{t+k}$ is the event that team i wins match $t+k$, ${}_iL_{t+k}$ is the event that team i loses match $t+k$, and H_t is the history of all matches played (in the tournament) up to time (match) t.

Note that the match number indexes time so that we can use the term match $t+k$ to mean the match played at time $t+k$. Where matches are played simultaneously (e.g. 3pm on a particular Saturday), matches can still be assigned a match number. We would only need to distinguish

matches that are played simultaneously with some further index variable. Definition (1) can be extended in a straightforward way to include the case where draws are possible. Thus, match *t+k* is unimportant if $\Pr({}_iX|{}_iW_{t+k} \cup {}_iD_{t+k}, H_t) - \Pr({}_iX|{}_iL_{t+k}, H_t) = 0$, where ${}_iD_{t+k}$ is the event that team *i* draws (ties) match *t+k*.

Extending the definition 1 to define a continuous measure of match importance, we arrive at the following.

Definition 2: The importance of match *t+k* to team *i* at time *t* with respect to outcome *X* is given by

$$ {}_iS(X)_{t,t+k} = \Pr({}_iX|{}_iW_{t+k}, H_t) - \Pr({}_iX|{}_iL_{t+k}, H_t)\,. \tag{1}$$

This is called the *conditional importance* of match *t+k* (for team *i* with respect to outcome *X*) by Schilling [6]. We might therefore term it the Schilling importance of match *t+k*. Morris [7] employed an analogous idea with regard to individual points in a game of tennis. Bojke [8] used a similar measure in his analysis of the effect of the introduction of play-offs for promotion to the FAPL.

Match *t+k* may effect the tournament outcome for a team other than that playing in the match—the importance measure, equation (1), is defined for all teams in the tournament, not just for those playing in the match of interest, match *t+k*. Extending the definition to include the cases of draws is problematic since necessarily a win as opposed to a draw for team *i* will have a greater effect on the tournament outcome—team *i* will be more likely to achieve *X* if they win as opposed to if they draw match *t+k*. Thus two separate importance measures might be proposed—win-importance and win-or-draw importance. These may be combined linearly into a single measure using suitably chosen weights. Choice of these weights is not obvious however—one way forward would be to determine the weights by the relative sizes of the win and draw probabilities for matches in general. In this paper, we will consider only wins as opposed to loses and thus use the Schilling importance of a match.

Other measures that have been proposed in the literature are simpler. Jennett [2] was the first to discuss the issue at length and proposed to measure the championship importance (importance of a match with respect to winning the tournament) as the inverse of the number of wins required from the remaining fixtures—Jennett calculates such a score for each team in each match in his analysis of attendance demand. His measure takes the value 0 when it is no longer possible for a team to win the championship. A similar measure is proposed when the outcome of interest is (avoiding) relegation from the league. Downward and Dawson [9] discuss a measure that is a refinement of Jennett's and depends on two quantities measured at the time of the match: the number of points that the team of interest is behind the league leaders, *b*; and the number of games left to play, *n*. However, it is not clear whether the authors are attempting to measure the importance of a match in determining the outcome of the tournament, or the level of uncertainty in the tournament outcome at the time of the match. The importance measure $I = (1+b)^{-\alpha} n^{-\beta}$, for suitably chosen α and β (>0), is in the spirit of that considered by them, with maximum value *I*=1 for *n*=1 and *b*=0, $I<1$ for $n>1$ or *b*>0, and $I \approx 0$ for *n* or *b* large. Audas *et al.* [10] define a match to be important (they use the term significant) if it is possible for either of the competing

teams to win the championship, be promoted or relegated if all other teams take 1 point each from each of the remaining matches. These measures recognize the time variation of match importance. Forrest *et al.* [4] use a measure which is the absolute difference in points per game for the two competitors in the match—this does not appear to distinguish mid-table from top or bottom of table matches nor take account of the time effect—matches closer to the end of season, if they can influence outcome, will have a greater effect on outcome. Thus the measures used to date have been ad hoc and empirical.

The Schilling importance is a general measure—although in practice one would need to estimate it. To estimate the Schilling importance of a match a probabilistic model for the results of the remaining matches in the tournament is required. Schilling [6] provides this in his paper for a small tournament (e.g. two teams playing a "best of seven" tournament) by assuming values for the win probabilities for the reference player in the remaining matches excepting the match of interest. We consider a probabilistic model for a larger tournament, but before we discuss such a model, we consider briefly the uses of a match importance measure.

3. Using match importance measures

To date, the principal use of a measure of match importance has been as an explanatory variable in econometric models applied in sport in order to consider questions of attendance demand and the determinants of this variable. A question of interest considered by, for example, Jennett [2] and Downward and Dawson [9] is: To what extent can the spectator demand for a match be explained by match outcome uncertainty and the importance of a match to the tournament outcome? Economists are interested in this question because the administrators of a sport are concerned with maximising the interest of the consuming public, and believe that competitive balance is important to the success of a tournament. Administrators would like to decide appropriate levels of regulation for their respective sport in order to maintain competitive balance. Other authors have used match importance measures as explanatory variables in models to predict match outcome (e.g. Audas *et al*. [10]; Morley and Thomas [11]).

Forrest *et al*. [4] suggest to use match importance to determine which matches to show when broadcasters are faced with constraints on the number of matches they can air and the number of times each team appears on air. They discuss the complex rules that apply to the FAPL. In particular, in the second half of the season, matches for broadcast are chosen two weeks in advance of their scheduled playing time—a suitable measure of match importance might be used to inform such decisions.

Decisions have to be made by sports administrators about the design of a tournament with regard to, for example, the number of teams competing, the rules for qualification or promotion and relegation or elimination, and changes to points schemes. Increasing match importance overall may be a driving factor—this could be particular argued when considering the introduction of play-offs for promotion. In this event, a suitable measure of match importance would be required. Tournament re-design would necessarily require the use of a number of suitable metrics—Scarf and Bilbao [5] discuss metrics for re-designing tournaments in a general context. Cairns [3] in particular uses a simple match importance measure to analyse the effect of re-organisation of the Scottish Football League in 1975, and demonstrates, as we might expect, that fewer teams in the

league tournament leads to a higher proportion of important matches. Match importance measures might be employed in a similar manner to consider changes of structure—moving from a single round-robin group-stage, to two round-robin group-stages as occurred in the UEFA Champions league. The ICC cricket world cup is another tournament that has also been subject to structural changes over its recent history.

We might suggest that competitors or teams use match importance to determine the effort that should be invested in a match. Players may wish to adapt their play in matches that proceed an important match, particular where there is a risk of suspension of a player for an important match in the near future for disciplinary reasons. Adaptive play might also be considered within a match, for determining those points in a match at which to increase or decrease effort. This has been suggested for the game of tennis (Morris [7]; Klaassen and Magnus [12]). Coaches could also use match importance in decision-making about team selection.

Fixture schedulers might wish to ensure that important matches for teams are spread evenly through the season—a suitable match importance measure would allow this to be done objectively. Finally, the best referees could be assigned to the most important matches.

4. Probabilistic models for match outcomes

The Schilling importance requires a probabilistic model for predicting the results of remaining matches in the tournament. Thus we need to specify a class of models and a set of potential predictor variables. To simplify this prediction problem, we assume that the results of matches are independent given the values of the predictors. Then we can use a univariate generalized linear model, and since outcomes are observed directly as win, draw or loss, it is natural to focus on models suitable for a categorical response variable. (For small round-robins and when within match events are used to separate competitors, indirect models that consider the bivariate scores (e.g. Maher [13]; Dixon and Coles [14]; Karlis and Ntzoufras [15]; McHale and Scarf [16]) can be used.) Many authors have used probit regression to model results directly (e.g. Dobson and Goddard [17]; Audas *et al.* [10]; Forrest *et al.* [4]). Typically, the match outcome, Y, is modelled as

$$Y = \begin{cases} 1\,(\text{win}) & \text{if } c_1 + \varepsilon \le \beta^T X, \\ 0\,(\text{draw}) & \text{if } c_{-1} + \varepsilon \le \beta^T X < c_1 + \varepsilon, \\ -1\,(\text{loss}) & \text{if } \beta^T X < c_{-1} + \varepsilon, \end{cases} \tag{2}$$

where $\beta^T X$ is a linear combination of predictor variables (the linear predictor), $c_1 + \varepsilon$ is a random cut-off point for winning with a systematic component c_1 and a random component $\varepsilon \sim N(0,1)$, and $c_{-1} + \varepsilon$ is a random cut-off point for losing. Thus Y has a multinomial distribution with three categories denoted by MN3($p_1, p_0, p_{-1}; p_1 + p_0 + p_{-1} = 1$) with

$$p_1 = \Pr(Y = 1) = \Pr(\text{win}) = \Phi(\beta^T X - c_1),$$

$$p_0 = \Pr(Y = 0) = \Pr(\text{draw}) = \Phi(\beta^T X - c_{-1}) - \Phi(\beta^T X - c_1),$$

$$p_{-1} = \Pr(Y = -1) = \Pr(\text{loss}) = 1 - \Phi(\beta^T X - c_{-1}) = \Phi(c_{-1} - \beta^T X).$$

where $\Phi(.)$ is the cumulative distribution function of the standard normal distribution, $N(0,1)$.

Home advantage may be considered as a fixed effect in the linear predictor or as a random effect through modification of the cut-off parameters, as in Koning [18].

Ordinal logistic regression is conceptually simpler and more natural in that the *log odds* of the ordered outcomes (in the sense that win, draw, loss form a natural ordering) are modelled as a linear function of predictor variables. This is sometimes referred to as the proportional odds model (McCullagh and Nelder [19], p.153). Cumulative probabilities of the ordered response are modelled with *logit* functions, so that

$$\log\left\{\frac{\Pr(\text{win})}{1-\Pr(\text{win})}\right\} = \log\left\{\frac{p_1}{1-p_1}\right\} = \alpha_1 + \beta^T X,$$

and

$$\log\left\{\frac{\Pr(\text{win or draw})}{1-\Pr(\text{win or draw})}\right\} = \log\left\{\frac{p_1+p_0}{1-p_1-p_0}\right\} = \alpha_0 + \beta^T X. \qquad (3)$$

It follows that

$$\Pr(\text{win}) = \text{logit}^{-1}(\alpha_1 + \beta^T X),$$

$$\Pr(\text{win or draw}) = \text{logit}^{-1}(\alpha_0 + \beta^T X),$$

where $\text{logit}^{-1}: \Re \to [0,1]$ such that $\text{logit}^{-1}(x) = \exp(x)/\{1+\exp(x)\}$. The logit function here is called the link function in the terminology of generalized linear models. Both Pr(win) and Pr(win or draw) increase with $\beta^T X$. We require $\alpha_1 > \alpha_0$ in general. From equation (3) we obtain

$$\log\left\{\frac{1-\Pr(\text{loss})}{\Pr(\text{loss})}\right\} = \alpha_1 + \beta^T X,$$

that is

$$\log\left\{\frac{\Pr(\text{loss})}{1-\Pr(\text{loss})}\right\} = -\alpha_1 - \beta^T X.$$

If we consider *Y*~MN3($p_1, p_0, p_{-1}; p_1 + p_0 + p_{-1} = 1$) then

$$\begin{aligned} p_1 &= \text{logit}^{-1}(\alpha_1 + \beta^T X), \\ p_0 &= 1 - \text{logit}^{-1}(\alpha_1 + \beta^T X) - \text{logit}^{-1}(-\alpha_0 - \beta^T X), \\ p_{-1} &= \text{logit}^{-1}(-\alpha_0 - \beta^T X). \end{aligned} \qquad (4)$$

Thus we have a model very similar to the ordered probit model. This is because $\Phi(.)$ and $\text{logit}^{-1}(.)$ are both functions $f: \Re \to (0,1)$ and

$$\Pr(\text{win}) = f(\alpha_1 + \beta^T X),$$

$$\Pr(\text{loss}) = f(-\alpha_0 - \beta^T X),$$

in both cases (with $\alpha_1 = -c_1$ and $\alpha_0 = -c_{-1}$). $\Phi^{-1}(.)$ serves as the link function in the ordered probit model. Furthermore, the ordinal logistic regression model may be derived by considering ε in equation (2) as having a standard logistic distribution (McCullagh and Nelder [19]). Specifying

other link functions gives rise to other models—the complementary log-log link $\text{clog}^{-1}:\Re \to [0,1]$ such that $\text{clog}^{-1}(x) = 1 - \exp(-e^{x})$ leads to the proportional hazards model (Cox [20]). This model may again be derived using the interval approach (equation 2) by considering ε as having an extreme value distribution. In spite of its similarity to probit regression and ease of interpretation, logistic regression is little used in soccer. When considering model-fit diagnostics, these two models are often indistinguishable. Scarf and Shi [21] used logistic to model the results of cricket matches. Nominal as opposed to ordinal multinomial models could also be considered for direct outcomes.

Now we consider the specification of suitable predictor variables. It is customary to use variables that represent the short term and long term strength of competitors—for example, Audas *et al*. [10] use the average points-ratio of each team/competitor (points per match) over the 12 months prior to the game of interest and over the period 12-24 months prior to the game of interest to represent long-term strength. They use the actual results of the immediately preceding *n* matches to represent short-term form—points per match in the last *n* matches would be simpler. Other variables may be included as candidate predictors such as the distance between the grounds of the home and away team, and a variable that indicates a competitor's involvement in another concurrent tournament. This approach is not straightforward however since to calculate the Schilling importance of a match at time *t* we must predict the results of all remaining matches in the tournament. Standard approaches, e.g. Audas *et al.* [10], use predictors that allow only one step ahead prediction—that is prediction of match *t*+1 given matches up to *t*. Two solutions to this problem are possible: 1) use only predictors for match *t*+*k* that are known at time *t*—this excludes the possibility of using short-term strength (form) variables; 2) use predictors for match *t*+*k* that may themselves be predicted at time *t*—form variables may then be used as predictors but predicted results will be based on predicted form! We recommend the latter in order to reflect variability in outcomes of matches that are some distance in to the future. Variables that are not time based e.g. geographical distance, are not precluded. Having chosen a set of potential predictors, a final subset of "best predictors" may be obtained using standard methods e.g. minimum AIC or stepwise regression.

5. Calculating the importance of a match

Analytical calculation of the importance, equation (1), is not possible except in the simplest cases—when only a small number of matches remain to be played. Therefore we resort to Monte Carlo simulation. This proceeds as follows. At time *t*, simulate random outcomes of the remaining matches in the tournament using the match outcome prediction model described above, conditional on: 1) a win for team *i* in the match of interest, match *t*+*k*; 2) a loss for team *i* in the match of interest, match *t*+*k*. This then provides single realisations of the indicator variables

$$I_{j,W} = \begin{cases} 1 & \text{if} \quad {}_iX|_i W_{t+k}, H_t \quad \text{occurs,} \\ 0 & \text{if} \quad {}_iX|_i W_{t+k}, H_t \quad \text{does not occur,} \end{cases}$$

and

$$I_{j,L} = \begin{cases} 1 & \text{if} \quad {}_iX|_i L_{t+k}, H_t \quad \text{occurs,} \\ 0 & \text{if} \quad {}_iX|_i L_{t+k}, H_t \quad \text{does not occur,} \end{cases}$$

That is, $I_{j,W}$ takes the value 1 if team i achieves the outcome of interest (winning the championship) conditional on winning the match of interest and the results of matches to date (time t), and 0 otherwise. Likewise for $I_{j,L}$. Repeating the simulation for j=1,…,N provides Monte Carlo "estimates"

$$f({}_iX|{}_iW_{t+k}, H_t) = \sum_{j=1}^{N} I_{j,W} \,/\, N$$

and

$$f({}_iX|{}_iL_{t+k}, H_t) = \sum_{j=1}^{N} I_{j,L} \,/\, N$$

of $\Pr({}_iX|{}_iW_{t+k}, H_t)$ and $\Pr({}_iX|{}_iL_{t+k}, H_t)$ respectively. These can then be substituted in equation (1) to obtain a Monte Carlo estimate of the Schilling importance, ${}_i\hat{S}(X)_{t,t+k}$. Care has to be taken with the simulations to ensure that random simulations of match outcomes are generated using equation (4); that is, generate an outcome at random from the distribution (4) rather than generating the most likely outcome as this latter outcome will remain fixed over repeated simulations.

6. Measuring match importance for the FAPL

We consider soccer in the English league in detail. In particular, we calculate match importance measures for the 2004/5 season of the Football Association Premier League (FAPL). This tournament is a home and away round-robin comprising 20 teams, beginning in August, with the winners crowned as Champions in May. The bottom three teams are relegated to a lower division at the end of the season and the top 4 teams qualify for a place in UEFA Champions League in the following season. We take January 1 2005 as the current time, t, at which point 200 of the 380 matches had been played. Our objective is to calculate the Schilling importance for a subset of remaining matches.

The match result prediction model is maintained to be relatively straightforward. We only consider, in the model building phase, short-term and long-term strength variables for home and away teams. The results of three seasons (2001/02, 2002/03, 2003/04) are used as training data for model fitting, and the results of season 2004/05 are reserved for examining the model fit. Table 1 shows the results of various models. Here, V^S is a season-long strength difference variable: the difference in points per match for the home and away teams based on all matches in the season to date. We define U_+^{HM} to be the points achieved by the home team in its last M home matches; U_-^{HM} to be the points conceded by the home team in its last M home matches; U_+^{AM} to be the points achieved by the away team in its last M away matches; U_-^{AM} to be the points conceded by the away team in its last M away matches. We set $U^{HM} = U_+^{HM} + U_-^{AM}$—this can be interpreted as the home strength and away weakness form, combined. Also, $U^{AM} = U_+^{AM} + U_-^{HM}$. These variables are in the spirit of the model of Maher [13] in which the home scoring rate is the product of home attack strength and away defensive weakness. These variables are considered for various M, M=1,3,5. Table 2 shows parameter estimates for the minimum AIC model and table 3 assesses the predictive power of the model—this is not too good although we can argue that this merely underlines the fact that match outcome uncertainty in soccer is high.

Figure 1 shows the estimated match importance for home and away teams for the remaining matches in the season measured at fixed time t=200 (at match 200) for each of the outcomes of interest. Small random perturbations have been added so that zeros (unimportant matches) may be distinguished in the plots; the proportion of matches that are unimportant with respect to the championship, European qualification, and relegation are 52, 14, and 14% respectively. The importance for a subset of these matches, with teams playing indicated, is shown in table 4. Figure 2 shows the match importance measure for fixed lag, k, for two cases, k=1,50. The percentage of unimportant matches has been calculated in each case. We can think of the case k=1 as short range forecasts of match importance, and k=50 as long range forecasts. For fixed lag, as t increases it is apparent that fewer matches are estimated to be important. In particular, figure 2(c) indicates that as the season progresses, relegation importance becomes more polarized—we would anticipate this since, as the season progresses, matches become either very important or not important at all. On the other hand, figure 1 indicates that for fixed t (looking forward from a fixed point in the season), there is little effect of lag on the estimated importance—while the importance of matches may become polarized as the season progresses, as k increases there will be more uncertainty in the measure of importance, and these two effects mitigate one another. It is interesting to note that no matches are estimated to be unimportant with respect to all outcomes when looking from a fixed point (t=200), while this figure is 10% when looking one step ahead (k=1)—these are interesting indicators of the effectiveness of the tournament design. Note that matches may be of little championship importance to the leading team when this team is a long way ahead—the league leader's losing one match can make little difference to its winning overall. It can be likewise for relegation. We have not considered the importance of matches to teams other than those playing in the match of interest, although this extension would be straightforward to carry out.

To investigate the effectiveness of the simulation, we consider the importance for a particular match (match number 217) as a function of the number of simulations (figure 3). The importance has been estimated and plotted in two cases: at time t=200 in figure 3(a); at time t=216 (lag 1) in figure 3(b). Quite a large amount of variability is evident here, with greater variability for the medium range forecast—this is as we might expect given the low predictive power of the match prediction model.

Finally, we might consider the financial implications for winning as opposed to losing particular match. The financial importance of match $t+k$ played at time t might be measured by

$$ {}_iS(F)_{t,t+k} = E({}_iF|_iW_{t+k}, H_t) - E({}_iF|_iL_{t+k}, H_t), $$

where $E({}_iF|_iW_{t+k}, H_t)$ is the expected financial reward achieved by team i at the end of the season given that team i wins match $t+k$ and given the history of all matches played up to time t. $E({}_iF|_iL_{t+k}, H_t)$ is likewise but given that team i loses match $t+k$. Table 5 shows this measure for matches 201-220. As future broadcasting income is not included here and because such income decreases significantly on relegation, it is evident that matches which influence qualification for Europe have the largest estimated financial importance. Note that small negative values are possible here due to the uncertainty of outcome in the Monte Carlo simulations. Also it is apparent from table 5 that the Schilling importance measures are not additive. The total importance of matches $t+k_1$ and $t+k_2$ is not equal to the sum of the importance of match $t+k_1$ and the

importance of match $t+k_2$—this is because the Shilling importance measures the effect of a single match based on the expected outcomes of matches not yet played. Figure 5 shows a histogram of this measure for the remaining matches estimated at t=200.

7. Discussion

We describe how a general measure of the importance of an individual match or tie in a tournament or contest may be defined and calculated. This measure has many uses, for example, in tournament design, in player and referee assignment, in fixture scheduling, and in broadcast scheduling. The measure is somewhat more difficult to compute than the ad hoc measures that have been used in practice to date. However, we argue that the measure we define has general applicability. In fact the definition may be extended to consider, in general, not just the importance of matches, but also the importance of points within matches or even the importance of events within matches. For example, the measure might be used, in principal, to measure the importance of ball by ball actions in cricket say, inning by inning events in baseball, or point by point actions in tennis. In this way, players may be able to determine, given information about important points in a match, when to increase their effort. Furthermore, if the importance of events, e.g. a run scored or a wicket taken in cricket, can be measured in this way, we might award points to players for actions in proportion to the importance of the action. Such a points scheme might form the basis of a player performance rating system.

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Table 1. Results of model fitting: log-likelihood and AIC for various models (subset of those fitted).

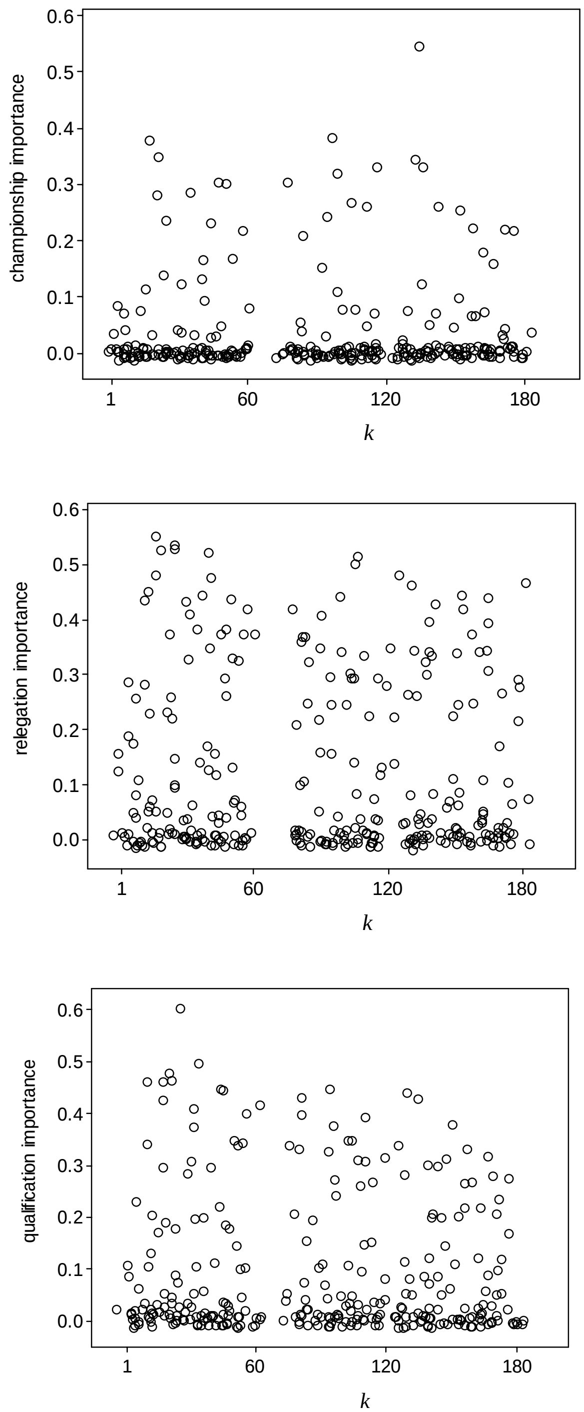
Predictors	No. of parameters	Log-Likelihood	AIC
$U^{A5}+V^{S}$	4	-989.37	**1986.7**
$U_{+}^{A5}+U_{-}^{H5}+V^{S}$	5	-989.14	1988.3
$U^{H5}+V^{S}$	4	-989.83	1987.6
$U^{A1}+U^{A5}+V^{S}$	5	-988.89	1987.8
$U^{H1}+U^{A5}+V^{S}$	5	-988.92	1987.8
$U^{H5}+U^{A5}+V^{S}$	5	-989.07	1988.1
$U^{H3}+U^{A5}+V^{S}$	5	-989.08	1988.2
$U^{A3}+U^{A5}+V^{S}$	5	-989.11	1988.2
$U^{H3}+U^{H5}+U^{A5}+V^{S}$	6	-988.43	1988.9
$U^{H1}+U^{H5}+U^{A5}+V^{S}$	6	-988.51	1989.0
$U^{HP5}+U^{A1}+U^{A5}+V^{S}$	6	-988.51	1989.0
$U^{H1}+U^{H5}+V^{S}$	5	-989.56	1989.1
$U^{A5}+V^{S}$ (nominal)	6	-989.23	1990.5

Table 2. Fitted parameter estimates for minimum AIC logistic regression model with the covariates U^{A5} and V^{S}, with standard errors in parenthesis, and p-values (observed significance level of test of parameter value equals zero).

		Z-values	P>|z|	95% Confident Interval	
β_1	-0.035 (0.017)	-1.98	0.047	-0.069	-0.000
β_2	0.344 (0.061)	5.66	0.000	0.225	0.464
α_1	-1.425 (0.209)				
α_{-1}	-0.234 (0.204)				

Table 3. Concordance table (cross-classification of observed and expected match outcomes) for validation data for ordinal logistic regression model with covariates U^{A5} and V^{S}.

		Predicted outcomes				
		-1 loss	0 draw	1 win	model percent correct	random guess % correct
Observed outcomes	-1 loss	29	26	36	32%	28%
	0 draw	24	28	46	29%	26%
	1 win	33	47	83	51%	46%
	overall percentage				40%	33%



(a)

(b)

(c)

Figure 1. FAPL 2004/5 season. Match importance for home and away teams measured at fixed time t=200 (at match 200) for remaining matches $t+k$, k=1,…,180; (a) championship outcome; (b) relegation outcome; (c) qualification outcome. Pertubations added to distinguish zeros.

Table 4. FAPL 2004/5 season. Estimated values of importance at time t (match 200) for a sample of 40 subsequent matches played in 4 blocks, for various outcomes: championship (C), European qualification (Q) and relegation (R). MI is the ordinal match identification number within the season 2004/2005.

MI	${}_1S(C)_t$	${}_2S(C)_t$	${}_1S(R)_t$	${}_2S(R)_t$	${}_1S(Q)_t$	${}_2S(Q)_t$	home team, 1	away team, 2
201	0.00	0.03	0.00	0.00	0.02	0.01	Liverpool	Chelsea
202	0.00	0.00	0.12	0.17	0.00	0.00	Bolton	WBA
203	0.00	0.08	0.01	0.00	0.06	0.22	Tottenham	Everton
204	0.00	0.00	0.28	0.17	0.00	0.00	Fulham	Crystal
205	0.00	0.00	0.07	0.19	0.02	0.00	Man City	Southampton
206	0.00	0.09	0.00	0.00	0.10	0.08	Charlton	Arsenal
207	0.00	0.00	0.06	0.01	0.01	0.02	Newcastle	Birmingham
208	0.00	0.00	0.05	0.12	0.00	0.00	Aston Villa	Blackburn
209	0.00	0.00	0.02	0.27	0.02	0.00	Portsmouth	Norwich City
210	0.01	0.05	0.00	0.00	0.35	0.10	Middlesboro	Man United
211	0.00	0.00	0.24	0.00	0.00	0.21	Blackburn	Charlton
212	0.00	0.00	0.43	0.25	0.00	0.02	WBA	Newcastle
213	0.00	0.00	0.56	0.06	0.00	0.02	Crystal	Aston Villa
214	0.00	0.00	0.46	0.00	0.00	0.29	Norwich	Liverpool
215	0.08	0.00	0.00	0.04	0.46	0.04	Everton	Portsmouth
216	0.00	0.00	0.06	0.06	0.02	0.01	Birmingham	Bolton
217	0.10	0.00	0.00	0.00	0.48	0.12	Man United	Tottenham
218	0.36	0.04	0.00	0.00	0.02	0.47	Chelsea	Middlesboro
219	0.28	0.00	0.00	0.02	0.18	0.05	Arsenal	Man City
220	0.00	0.00	0.47	0.27	0.00	0.00	Southampton	Fulham

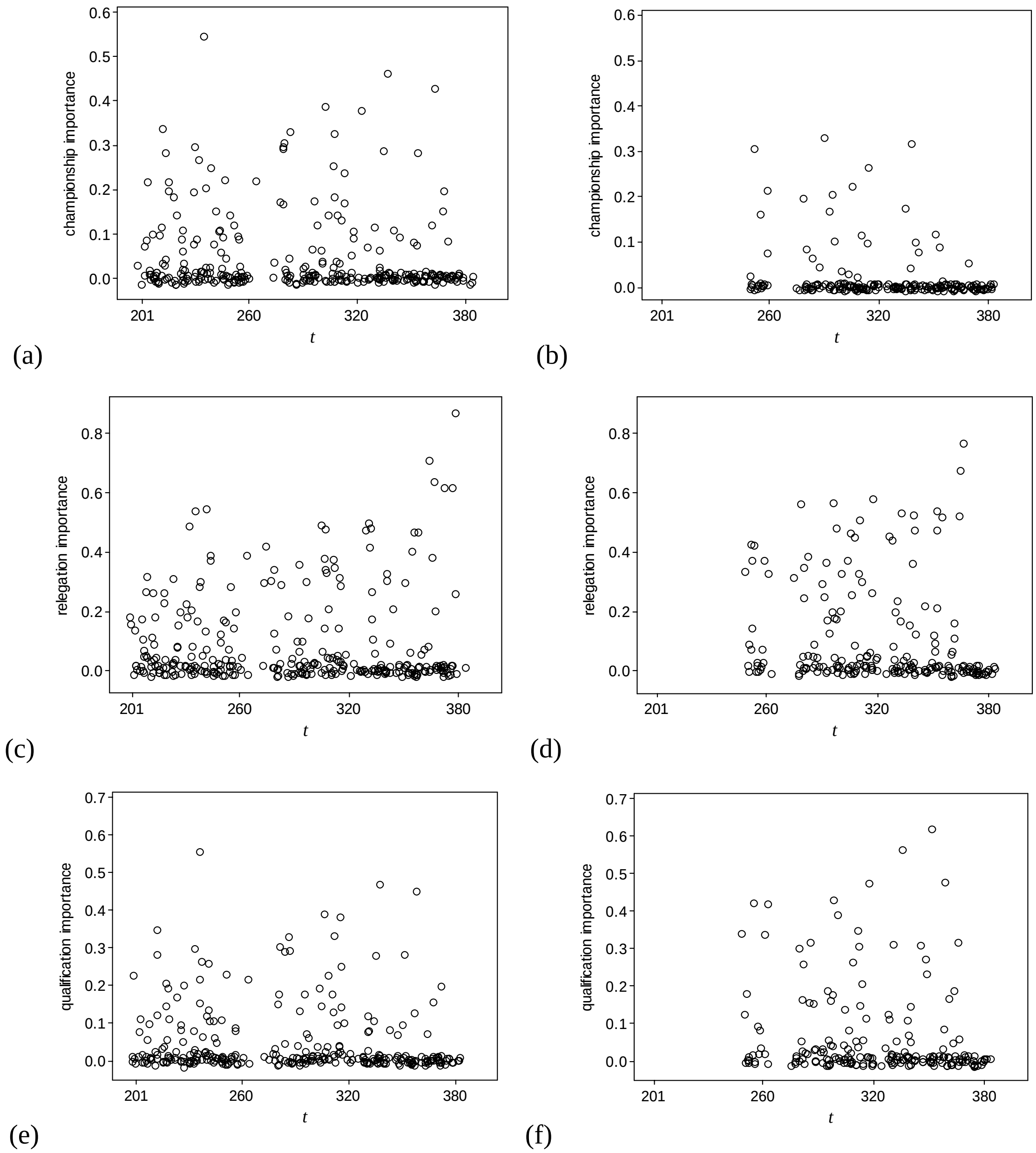
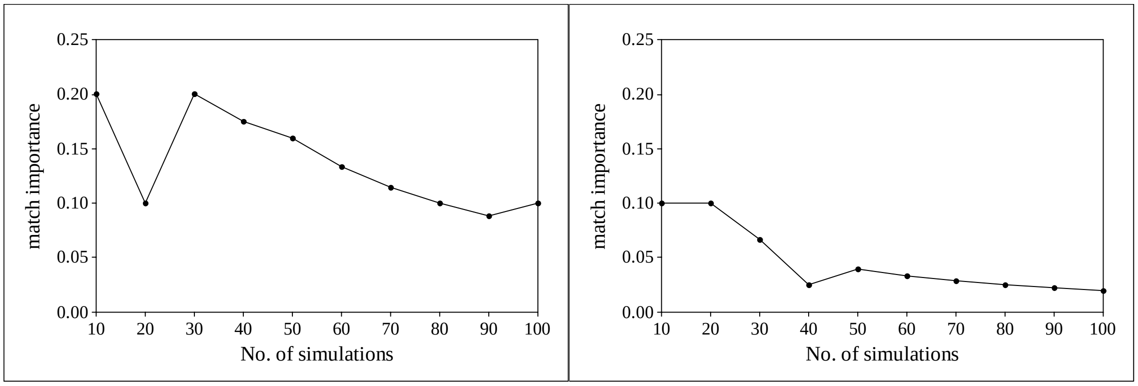


Figure 2. FAPL 2004/5 season. Match importance for home and away teams measured at fixed lag k for remaining matches $t+k$, t=200,…,379; (a) championship outcome, k=1, 72% unimportant; (b) championship outcome, k=50, 71% unimportant; (c) relegation outcome, k=1, 14% unimportant; (d) relegation outcome, k=50, 29% unimportant;(e) qualification outcome, k=1, 36% unimportant; (f) qualification outcome, k=50, 34% unimportant. Pertubations added to distinguish zeros.



(a) (b)

Figure 3. Estimated championship importance for the home team of match 217 (Man United v Tottenham on 4th January 2005) against the number of simulation repetitions: (a) current time t=200, ${}_1\hat{S}(C)_{200,217}$ plotted; (b) current time t=216, ${}_1\hat{S}(C)_{216,217}$ plotted.

Table 5. FAPL 2004/5 season. Values of financial importance measure at time t (match 200) for a sample of 20 subsequent matches played in 2 blocks. Financial reward takes account of prize money for FAPL, likely UEFA Champions League and UEFA Cup earnings but not income resulting from broadcast rights. Monetary unit is £10^6.

MI	${}_1S(F)_t$	${}_2S(F)_t$	home team, 1	away team, 2
201	-0.02	0.16	Liverpool	Chelsea
202	1.03	0.58	Bolton	WBA
203	1.80	3.67	Tottenham	Everton
204	0.85	0.44	Fulham	Crystal
205	1.67	0.63	Man City	Southampton
206	2.52	1.40	Charlton	Arsenal
207	1.05	1.64	Newcastle	Birmingham
208	1.33	0.80	Aston	Blackburn
209	1.31	0.76	Portsmouth	Norwich
210	6.17	1.45	Middlesboro	Man United
211	1.71	4.90	Blackburn	Charlton
212	1.06	2.48	WBA	Newcastle
213	1.61	2.24	Crystal Palace	Aston Villa
214	1.67	5.68	Norwich City	Liverpool
215	7.23	2.59	Everton	Portsmouth
216	2.96	1.79	Birmingham	Bolton
217	7.30	4.05	Man United	Tottenham
218	0.59	8.25	Chelsea	Middlesboro
219	2.99	2.67	Arsenal	Man City
220	1.52	1.42	Southampton	Fulham

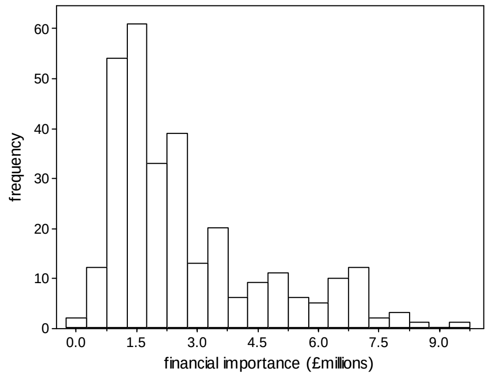


Figure 4. Histogram of financial importance of remaining matches in 2004/5 season estimated at match 200 (t=200).